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Defining the Abatement Cost in Presence of Learning-by-doing: Application to the Fuel Cell Electric Vehicle.

Anna Créti[1], Alena Kotelnikova[2], Guy Meunier[3], Jean-Pierre Ponssard.[4]

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Contact: guy.meunier@ivry.inra.fr

[1] Ecole Polytechnique, Universit́ e Paris Dauphine PSL
[2] Ecole Polytechnique
[3] Ecole Polytechnique, INRA-UR1303 ALISS
[4] Ecole Polytechnique, CESifo, CIRANO and CNRS

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Defining the Abatement Cost in Presence of Learning-by-doing: Application to the Fuel Cell Electric Vehicle. *

July 2017

Anna Creti[a,b], Alena Kotelnikova[a], Guy Meunier[a,c†], Jean-Pierre Ponssard[a,d]

[a] Ecole Polytechnique

[b] Université Paris Dauphine PSL

[c] INRA-UR1303 ALISS

[d] CESifo, CIRANO and CNRS

Abstract

We consider a partial equilibrium model to study the optimal phasing out of polluting goods by green goods. The unit production cost of the green goods involves convexity and learning-by-doing. The total cost for the social planner includes the private cost of production and the social cost of carbon, assumed to be exogenous and growing at the social discount rate. Under these assumptions the optimization problem can be decomposed in two questions: (i) when to launch a given schedule; (ii) at which rate the transition should be completed that is, the design of a transition schedule as such. The first question can be solved using a simple indicator interpreted as the MAC of the whole schedule, possibly non optimal. The case of hydrogen vehicle (Fuel Cell Electric Vehicles) offers an illustration of our results. Using data from the German market we show that the 2015-2050 trajectory foreseen by the industry would be consistent with a carbon price at 52€/t. The transition cost to achieve a 7.5 M car park in 2050 is estimated at 21.6 billion € that is, using 4% discount rate, 115 € annually for each vehicle which would abate 2.18 tCO_2 per year.

JEL Classification: Q55, Q42, C61

Keywords: Dynamic abatement costs; learning by doing; fuel cell electric vehicles

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†Corresponding author, e_mail: guy.meunier@inra.fr, INRA, 65 bd de Brandebourg, 94200 Ivry-sur-Seine, France.

1 Introduction

Marginal abatement costs (MACs) are practical indicators used in policy discussions. Simply stated, the MAC is the marginal cost incurred for avoiding one unit of CO_2 emission through the substitution of a dirty technology by a clean technology. MACs are notably used to critically assess decarbonization efforts among sectors and arbitrage the opportunity of launching technical options (Marcantonini and Ellerman, 2014; Archsmith et al., 2015). This indicator is formally defined in a static context. The objective of this paper is to propose a simple extension in dynamic situations involving learning-by-doing effects.[1]

We consider a partial equilibrium model. The market size is fixed and initially served by polluting goods that must be produced at every point in time at constant marginal cost. The production cost of green goods at a given time is convex with respect to current production and decreases with cumulative past output thanks to learning-by-doing. The total cost of transition for the social planner, to be minimized, includes the private cost of production and the social cost of carbon. The latter is assumed to be exogenous and to grow at the social rate of discounting.

Under these assumptions we show that the optimization problem can be nicely decomposed into two questions: (i) when to launch a given schedule; (ii) at which rate the transition should be completed, that is, the design of a transition schedule as such. Interestingly the first question can be answered also for suboptimal schedules. Indeed we define a *dynamic abatement cost* (to be denoted as DAC) for a given transition schedule, possibly suboptimal. Its expression is quite simple to obtain thanks to our assumption regarding the evolution of the CO_2 price. The DAC can be interpreted as the MAC of the whole transition schedule (Proposition 1). As for the second question we show that the shape of the optimal transition schedule is independent of the CO_2 price level (Lemma 3). More precisely the CO_2 price sequence only determines the optimal launching date but not the optimal shape as such (Proposition 2). A comparative static property follows (Corollary 2). For completeness we also show how the DAC should be modified if the CO_2 grows at a rate different form the

[1] A survey of learning-by-doing rates for different energy technology can be found in IEA (2000) and McDonald and Schrattenholzer (2001). Learning rates (cost reduction when production doubles) varies from 25% for photovoltaics, 11% for wind power, and 13% for fuel cell in the period 1975-2000.

social discount rate (Corollary 3).

From a theoretical standpoint our model can be seen as a particular specification of similar models developed in the literature. It delivers the standard result that along the optimal trajectory the CO2 price should be equal to the sum of two terms: the difference between the cost of the marginal green good and a polluting good; and the learning benefits over the future (lemma 1). This result, well known in the literature on climate policy and induced technical change (e.g. Goulder and Mathai, 2000; Bramoullé and Olson, 2005) illustrates the inter-temporal consistency of the optimal trajectory.

The role of convexity has been stressed by Bramoullé and Olson (2005) in their study of the role of learning-by-doing in sectoral arbitrage. Without convexity, learning-by-doing alone does not justify a progressive deployment of the clean option and learning would occur in one single step. This feature is also present in our model (lemma 2). Our model is also reminiscent of researches on investment in clean capital with adjustment cost (e.g. Vogt-Schilb et al., 2012; Amigues, Lafforgue and Moreaux, 2015; Amigues, Ayong Le Kama and Moreaux, 2015). With learning-by-doing, clean units play a dual role of being both a consumption good and an investment in clean (knowledge) capital. These models typically recommend early deployment of green technologies. The recommendation of early deployment is also present in analytical macro models though these models remain imprecise on the specific sectoral cost assumptions that would justify their conclusions (e.g. Grimaud and Rouge, 2008; Acemoglu et al., 2012). All these models are difficult to implement. In particular they stress that levelized cost could be misleading to compare sectoral options. Our contribution provides conditions to overcome this difficulty through a simple indicator to assess the optimal launching time of the green technology.

The case of hydrogen vehicle (a.k.a. Fuel Cell Electric Vehicles) for the substitution of the mature gasoline vehicle (a.k.a. Internal Combustion Engine) provides an illustration for our methodology. We define a deployment schedule for hydrogen from 2015 to 2050, construct its associated yearly cost, compare this cost with the cost of ICE, and derive the dynamic abatement cost. Depending on the selected estimate of the social cost of carbon in 2015, this analysis provides some rationale for initiating the deployment of hydrogen in 2015 or later. Our data is mostly built from a study sponsored by industry (McKinsey & Company, 2010). There are a number of other studies on the deployment of hydrogen. We give special attention to Rösler et al. (2014) which carries an in depth investigation using the energy bottom-up model TIAM-ECN (Loulou, 2008; Loulou and Labriet, 2008) to build scenarios up to 2100 for passenger car transportation in Europe. They show that hydrogen vehicle could achieve most of the market in 2050 if no significant breakthrough in battery is made. Bruegel and the European School for Management and Technology revisit the economic rationale for public action for hydrogen vehicle (Zachmann et al., 2012). Harrison (2014) provides an extensive analysis of the environmental and macroeconomic impacts (growth, employment, trade) of alternative motor ways (gasoline, battery electric, hydrogen) at the European level at the 2050 horizon. In these models the total cost of ownership (TCO i.e. purchase price, maintenance and fuel, all costs annualized) for all power-trains are expected to converge around 2040. In contrast, Oshiro and Masui (2014) analyze the Japan market and find that battery electric vehicles would take most of the passenger car market in 2050 while the share of hydrogen would remain marginal.

Using our methodology we show that for the deployment of hydrogen vehicles proposed by industry (McKinsey & Company, 2010) for the German market would be consistent with a carbon price at 52 € /tCO_2. This is much lower than estimates obtained through a static approach which range typically around 800 to 1000 € /tCO_2 (e.g. Beeker, 2014). In a static approach the abatement cost decreases annually which pervades its interpretation. In our approach, we aim at the substitution of 7.5 M gasoline vehicles by hydrogen vehicles over a period of 35 years (2015 to 2050). At first the manufacturing cost of a hydrogen vehicle is 60 k € much higher than the manufacturing cost of a gasoline vehicle 22 k €. The learning-by-doing and the cumulative production of 13.4 M hydrogen vehicles implies that the manufacturing cost will drop to 22.8 € so that the total cost of ownership of both technologies will approximately converge in 2050. The discounted transition cost of this transition is estimated at 21,6 bn€ that is 2 882 € per vehicle in the 7.5 M car park, or 115 € per year assuming a 4 % discount rate. The substitution of polluting by greener cars will permanently abate 2.18 tCO_2 per vehicle thanks to the progressive introduction of carbon free technologies to produce hydrogen. This gives the 52 € /tCO_2 estimate.

The paper is organized as follows. Section 2 presents the analytical model and develops the first best and second best scenarios. Section 3 illustrates the application to the case of hydrogen versus gasoline, whereas Section 4 concludes.

2 The analytical framework

2.1 The model

We consider a simple model of a sector, say the car sector, whose size is constant. There are two varieties of vehicles: cars build by using an old polluting technology (gasoline vehicle) and new ones which are carbon-free (hydrogen vehicle). The new technology displays learning-by-doing.

Time is continuous from 0 to $+\infty$. The discount rate is constant equal to r. We consider that cars last one unit of time. There are N cars among which x new "green" cars and $N-x$ polluting old cars. Units are normalized so that each old car emits one unit of CO_2, green cars do not pollute. The cost of an old car is constant: c_o. The cost of x new green cars is a function of the knowledge capital X: $C(X,x)$. At any time $t \in [0,+\infty)$ the knowledge capital X_t is equal to the total quantity of green cars previously built $X_t = \int_0^t x_u du$.

The cost $C(X,x)$ is assumed thrice differentiable and positive. It is null for $x=0$, for all X: $C(X,0)=0, \forall X$, and strictly positive otherwise. The cost is increasing and convex with respect to x, the quantity of green cars produced. Knowledge reduces production cost and this effect decreases with the knowledge stock. The marginal production cost also decreases with knowledge. These assumptions translate formally as follows:[2]

$$C_x \geq 0, C_X \leq 0, C_{xx} > 0, C_{XX} > 0 \text{ and } C_{Xx} \leq 0. \tag{A1}$$

To ensure the convexity of the problem we assume that the following condition holds:

$$[C_{Xx}]^2 < C_{XX}C_{xx}. \tag{A2}$$

Finally, we assume that the effect of knowledge on the marginal cost, $-C_{Xx}$, is larger for larger production. Said differently, the derivative C_X is concave with respect to x, that is, $C_{Xxx} < 0$.

[2]Partial derivatives are denoted with indexes (except if it could be confusing) for instance C_X stands for $\partial C/\partial X$.

These cost assumptions are consistent with empirical observations for the deployment of hydrogen cars (McKinsey & Company, 2010; Brunet and Ponssard, 2016). Convexity matters at the beginning of the deployment and corresponds to exogenous upper bounds on investment rates. Infrastructure costs at the early deployment phase are an important factor that drives convexity during that phase. Here it is materialized by the assumption that $C_{xX} < 0$. Learning-by-doing matters as the cumulative production of green cars increases, as is commonly assumed in the deployment of green technologies. Learning-by-doing impacts the whole deployment and leads to time arbitrage (more production and higher expenses today for lower unit cost and lower expenses later). Its impact declines over time and is materialized by the assumption that the marginal production cost also decreases with knowledge.

The price of CO_2, or *social cost of carbon*, is p_t^{CO2}. It is exogenous, which is consistent with our partial equilibrium approach, and grows at the discount rate $p_t^{CO2} = e^{rt}p_0$ with $p_0 > 0$. This assumption will prove very useful to simplify dynamic considerations. It means that once discounted a ton of CO_2 emissions has the same value whatever the date at which it is emitted. Such a price dynamics could be linked to the stock nature of CO_2 emissions and low decays of CO_2 in the atmosphere. It would occur, in a dynamic model if there was a constraint on the total cumulated emissions according to a CO_2 Hotelling's rule (e.g. Vogt-Schilb et al., 2012; Schennach, 2000).[3]

Indeed, in an standard welfare framework the social cost of CO_2, its Pigouvian price, is equal to the discounted flow of damage from a current marginal ton of CO_2.[4] In practice it depends on various policy considerations, and numerical assessments of the optimal CO_2 price level and dynamics are debated.[5] The impact of the growth rate of the CO_2 price is

[3] According to the IPCC (2013) such a "Carbon Budget" is required to enforce a peak temperature target (see also Allen et al., 2009; Matthews and Caldeira, 2008).

[4] See Tol (2014) for a textbook derivation with several objectives. A targeted stabilization of the concentration of carbon in the atmosphere implies a growth rate of the CO_2 price equal to the interest rate plus the decay rate of carbon in the atmosphere (Tol, 2014, p. 53, the "shadow price" of CO_2 μ should be divided by $U'(C_t)$ to get the expression of the current price of CO_2).

[5] Several recent contributions attempt to provide and test rule of thumb pricing. Golosov et al. (2014) propose a simple formula, based on a specified dynamic general equilibrium framework: the price of carbon should be proportional to world gross domestic product (see also Grimaud and Rouge, 2014; Rezai and

discussed in sub-section 2.3.

A notation that will prove useful is the discounted cost of a fully green fleet with a initial knowledge stock X. This cost is the discounted sum of the costs of producing N green cars at each time. With this production schedule the knowledge stock at time t is $X + tN$ so this discounted cost, denoted $\Omega(X)$, is as follows:

$$\Omega(X) = \int_0^{+\infty} e^{-rt} C(X + tN, N) dt. \tag{1}$$

Altogether the objective of the social planner is to minimize the cost:

$$\Gamma = \int_0^{+\infty} e^{-rt} \left[(p_t^{CO2} + c_o).(N - x_t) + C(X_t, x_t) \right] dt \tag{2}$$

$$\textit{subject to}$$

$$\dot{X}_t = x_t, \ X_0 = 0 \tag{3}$$

$$0 \leq x_t \leq N \tag{4}$$

This is a standard problem, and the qualitative properties of the optimal trajectory have been already analyzed elsewhere, if not in the precise same framework in relatively similar ones. The following Lemma describes the features of the optimal trajectory.

Lemma 1 *The production of green cars increases over time. There are two dates T_s and T_e, with $T_s \leq T_e$, at which the transition respectively starts and ends, denoting x_t^* and X_t^* the optimal trajectory:*

$$x_t^* = 0 \textit{ for } t \leq T_s$$

$$0 < x_t^* < N \textit{ for } T_s < t < T_e$$

$$x_t^* = N \textit{ for } t \geq T_e$$

During the transition, that is, for $t \in (T_s, T_e)$ the following equation holds:

$$p_t^{CO2} = \underbrace{[C_x(X_t^*, x_t^*) - c_o]}_{\textit{static abatement cost}} + \underbrace{\int_t^{+\infty} e^{-r(\tau - t)} C_X(X_\tau^*, x_\tau^*) d\tau}_{\textit{learning benefit } (<0)} \tag{5}$$

van der Ploeg, 2015; Van den Bijgaart et al., 2016).

The proof is in Appendix A, the main step consists in proving that x_t is increasing. The optimal trajectory is a smooth transition in which green cars progressively replace old cars. At the end of the transition the fleet is completely green. During the transition, the green share of the fleet is determined by equation (5). This equation stating that the marginal cost should be equal to the marginal revenue plus the learning benefits recalls a well-known result in the literature on learning-by-doing whether related to climate policy (e.g. Goulder and Mathai, 2000; Bramoullé and Olson, 2005), or not (see Rosen, 1972 for an early discussion; and the survey by Thompson, 2010, eq. 3 p. 435):

Indeed, the static marginal abatement cost, that is, the difference between the cost of the marginal green car and an old car, is not sufficient to determine the optimal number of green cars as a function of the CO_2 price. One should also compute the learning benefits, that is, the reduction of future cost due to the production of one more green car today. The "relevant " marginal abatement cost, the right hand side of equation (5), cannot be computed without knowing the whole future optimal path of production, which limits the practical use of equation (5).

Considering two extreme cases is useful to interpret the role of our assumptions on cost in the time dependency. On the one hand, without learning-by-doing, the static abatement cost is sufficient to determine the optimal number of green cars at each date. There is a smooth transition. Still, each date can be isolated from the rest of the trajectory: there is no interdependency between past, present and future decisions. On the other hand, without convexity, the transition takes place at once and its date can be determined through some generalization of the notion of abatement cost. Learning-by-doing alone, instead, does not imply a smooth transition. This is pointed out below.

Lemma 2 *If $C_{xx} = 0$ then the optimal strategy is to replace all old cars by new cars from a date $T_s = T_e$. At this date the CO_2 price is:*

$$p_{Ts}^{CO2} = \frac{r\Omega(0) - c_o N}{N}. \tag{6}$$

in which $\Omega(0)$ is given by equation (1).

The proof is in Appendix B. The threshold of CO_2 price in the equation (6) could be interpreted as a MAC for the whole technical option: it is the ratio of the difference between the levelized (static) cost of a fleet of green cars and a fleet of old cars to the quantity of emissions abated by the project. In the next subsection, this rule is extended to a general cost function and a ramping deployment schedule.

2.2 The "deployment" perspective

In the optimal case, the whole trajectory is consistent with the CO_2 price: the date at which the deployment starts and the pace at which green cars replace old cars are jointly determined. For a real world application, this theoretical analysis does not provide a simple rule to evaluate a technical option and a "launching date". Furthermore, in real world issues there are many components in the cost (e.g. investment in infrastructure) that do not easily translate into a clear specification of $C(X, x)$ so that the determination of an optimal trajectory may be out of reach. It is more useful to discuss suboptimal trajectories.

We propose to decompose the global problem into sub-problems easier to connect to practical examples, offering straightforward interpretations. We disentangle the choice of the production schedule of cars during the deployment phase from the choice of a date at which deployment should start (the date T_s in the optimal scenario). More precisely, the global problem could be decomposed as follows. There is a "deployment schedule" of the green option with a finite duration; during this deployment an exogenous given amount of green cars is produced each year. The "launching date" of this deployment should be determined. Once deployment is achieved, the whole fleet is replaced by green cars.

Consider a given deployment schedule, possibly non optimal, the only variable to be chosen is the launching date T_l that should balance the price of CO_2 with the abatement cost of the deployment. Waiting one year to launch the deployment increases emissions by an amount proportional to the fleet but postpones the costly deployment and implementation of the green fleet. The discounted cost of the fleet, given by equation (2), can be decomposed to reflect this trade-off. To do so, the costs of the deployment and the fully green fleet should be discounted to be independent of the launching date:

- The deployment schedule takes place over D years during which a total quantity of $\bar{X}$ green cars are produced. The production schedule is denoted $\xi = (\xi_\tau)_{\tau \in [0,D]}$ in which

ξ_τ is the number of green cars produced at stage τ of the deployment (i.e. τ years after the launching), and $\bar{X} = \int_0^D \xi_\tau d\tau$. The cost of this deployment is the cost to produce ξ green cars instead of ξ old cars:

$$I((\xi_\tau)_{\tau\in[0,D]}) = \int_0^D e^{-r\tau}[C(\bar{X}_\tau, \xi_\tau) - c_o\xi_\tau]d\tau \text{ in which } \bar{X}_\tau = \int_0^\tau \xi_u du. \quad (7)$$

- At the end of the deployment the fleet is completely green, and the discounted cost of the green fleet of cars solely depends on $\bar{X}$, the knowledge accumulated during the deployment. This cost, viewed from the date of the end of the deployment, is $\Omega(\bar{X})$ given by equation (1).

The discounted cost of the fleet, given by equation (2), can then be written (with a slight abuse of notation):

$$\Gamma(T_l, \bar{X}, D, \xi) = \underbrace{\int_0^{T_l+D} e^{-rt}(c_o + p_t^{CO2})Ndt}_{\text{fully old fleet}} + \underbrace{e^{-rT_l}I(\xi) - p_0\bar{X}}_{\text{deployment phase}} + \underbrace{e^{-r(T_l+D)}\Omega(\bar{X})}_{\text{fully green fleet}}. \quad (8)$$

This cost is the sum of three terms: the cost, including the CO_2 cost, of a fleet of old cars from today to the end of the deployment; the cost of the deployment minus the gain from abatement during deployment; once deployment is achieved the fleet is entirely green and the current cost of the fully green fleet only depends on the quantity of knowledge accumulated during the deployment.

The problem is now simply to determine the date T_l at which the deployment should be launched. The assumption that the CO_2 price grows at the interest rate plays a key role here because the precise date at which carbon reduction takes place does not impact welfare. The emissions abated during deployment, which are precisely equal to the quantity of cars accumulated during the deployment, do not impact the choice of the launching date. This nicely fits our decomposition, since the choice of the launching date only changes costs via discounting.

Looking for a simple metric for the whole deployment several possibilities arise. While it is quite straightforward to compute the relative cost of the whole deployment, it is less clear how the CO_2 avoided during and after deployment should be aggregated. For a standard project (e.g. a wind farm), in a stable environment, the MAC indicates both the cost per ton of CO_2 avoided and the CO_2 price at which the project should be undertaken. Our approach build on this second consideration to define the Dynamic Abatement Cost.

Proposition 1 *The optimal launching date T_l^* of the deployment schedule $(\bar{X}, D, \xi)$ is such that the CO_2 price at the end of the deployment is equal to the Dynamic Abatement Cost of the deployment schedule:*

$$p^{CO2}_{T_l^*+D} = DAC\left(\bar{X}, D, \xi\right) =_{def} \frac{rI}{N}e^{rD} + \frac{r\Omega(\bar{X}) - c_0 N}{N}. \tag{9}$$

Proof. Taking the derivative of the discounted total cost Γ given by (8) with respect to the launching date gives:

$$\begin{aligned}\frac{\partial \Gamma}{\partial T_l} &= e^{-r(T_l+D)}(c_o + p^{CO2}_{T_l+D})N - rIe^{-rT_l} - r\Omega(\bar{X})e^{-r(T_l+D)} \\ &= e^{-r(T_l+D)}\left[p^{CO2}_{T_l+D}N + c_oN - rIe^{rD} - r\Omega(\bar{X})\right].\end{aligned}$$

At the optimal launching date this derivative is null and, consequently, equation (9) is satisfied. ■

This rule can be easily interpreted: the launching date is chosen so that the abatement cost of the whole deployment is equal to the CO_2 price at the end of the deployment. The abatement cost of the project is the sum of two components: the sunk cost of the deployment that takes D years (rI/Ne^{rD}); and the relative over-cost of a green car at the end of the deployment $((r\Omega(\bar{X}) - c_0)/N)$. The cost $r\Omega(\bar{X})$ is the annualized cost of a fully green fleet, so $r\Omega(\bar{X})/N$ is the average current cost of a green car over the life of the green fleet.

This suggests a simple interpretation of the DAC. Consider the deployment as the transition needed to achieve the *time to market* for the green technology. In that case, at the end of the deployment the cost of a green car would be stable, $(C(X, x) = \underline{c}x)$. The second component would then be the difference between the cost of a green and an dirty car $(\underline{c} - c_0)$ divided by the emissions saved by this substitution, that is the MAC at that time. Our definition of DAC appears as a generalization of the MAC; they coincide whenever there are no learning-by-doing gains in a deployment while, whenever there are, the first term represents the unit learning cost to achieve the time to market via the deployment of $\bar{X}$ to substitute

N units, again divided by the emissions saved by this substitution. If the time to market means that the green technology is competitive with the dirty technology, the second term vanishes and only the transition cost remains. Our paper provides clear assumptions that make this simple generalization meaningful and details how to compute the corresponding indicator.

One can also consider the CO_2 price at the launching of the deployment. Policy makers would certainly be more interested in this indicator: it gives when the trajectory is worth launching. It is obtained by discounting the DAC:

$$p_{T_l^*}^{CO2} = \frac{rI}{N} + \frac{r\Omega(\bar{X}) - c_0 N}{N} e^{-rD}. \tag{10}$$

Note that the price obtained in Lemma 2 corresponds to the price of Proposition 1 for an extreme deployment schedule in which there is no spreading of the cumulative production, that is when $\bar{X} = 0$ and $D = 0$, we have that $I = 0$.

Corollary 1 *Consider two cost functions $C_1(X, x)$ and $C_2(X, x)$. If for all X and x, the cost function 1 is lower than the cost function 2, then for any given deployment schedule $(\xi_\tau)_{\tau \in [0,D]}$, the optimal launching date comes earlier with C_1 than with C_2.*

The proof of this corollary is straightforward. For a given schedule, both the investment cost and the cost of a fully green fleet are lower with the cost C_1 and the associated launching date should then be earlier.

A more demanding result would be that an increase in the learning rate induces an earlier launching date. It does not hold in general since the optimal schedule depends on the learning rate and Corollary 1 cannot be applied. It holds in the extreme case of no convexity: then the deployment schedule is independent of the learning rate (Lemma 2).

2.3 The optimal trajectory revisited

The deployment approach can be used to recover the optimal trajectory described in Lemma 1. In the previous section we studied the optimal launching of a suboptimal deployment schedule which helps define the DAC. In this section, we will show that the DAC can be used to optimize the deployment schedule independently of the CO_2 price, proceeding by steps and further decomposing the global problem. This procedure will provide intuitive conditions on the characteristics of the deployment schedule and a criterion to evaluate small changes of a given deployment schedule. It highlights that the influence of the CO_2 price p_0 on the optimal trajectory is solely a matter of timing: a change of the CO_2 price modifies the launching date but not the optimal deployment schedule.

A deployment schedule is defined by $(\bar{X}, D, \xi = (\xi_\tau)_{\tau\in[0,D]})$ and together with the launching date T_l it gives a trajectory $(x_t)_{t\in[0,+\infty)}$. The optimal deployment schedule to be denoted $(\bar{X}^*, D^*, \xi^*)$ can be described with the optimal trajectory defined in Lemma 1: $\bar{X}^* = X_e^*$, $D^* = T_e - T_s$ and $\xi^* = (x^*_{T_s+\tau})_{\tau\in[0,D]}$ which is optimally launched at $T_l^* = T_s$. This optimal deployment schedule minimizes the total discounted cost Γ given by equation (8). This cost may be rewritten using the definition of the DAC (eq. (10)) and setting $T_e = T_l + D$:

$$\begin{aligned}\tilde{\Gamma}(T_l, \bar{X}, D, \xi) &= \int_0^{T_l+D} e^{-rt}(c_o + p_t^{CO2})N dt - p_0\bar{X} + e^{-r(T_l+D)}\left[e^{rD}I(\bar{X}, D, \xi) + \Omega(\bar{X})\right] \\ &= \frac{c_0 N}{r} + p_0(NT_e - \bar{X}) + e^{-rT_e}\frac{N}{r}DAC(\bar{X}, D, \xi). \end{aligned} \tag{11}$$

This decomposition suggests to proceed by steps by first minimizing the deployment cost of a total quantity $\bar{X}$ over D years and then optimally choosing these two aggregate characteristics.

Lemma 3 *For any triple T_l, $\bar{X}$ and D the schedule ξ that minimizes the discounted cost $\Gamma(T_l, \bar{X}, D, \xi)$, given by eq. (8), is independent of the CO_2 price and the launching date. This schedule minimizes the DAC subject to $X_D = \bar{X}$.*

Once the objective to produce $\bar{X}$ over D years is given, the DAC to do so should be minimized, which is equivalent to minimizing the deployment cost I. Let us denote $I^*(\bar{X}, D)$ the minimum deployment cost:

$$I^*(\bar{X}, D) = min_{\xi_\tau} \int_0^D e^{-r\tau}[C(X_\tau, \xi_\tau) - c_0\xi_\tau]d\tau \tag{12}$$

$$\text{s.t. } \dot{X}_t = \xi_\tau;\ 0 \le \xi_\tau \le N \text{ and } X_D = \bar{X}.$$

and $DAC(\bar{X}, D) = r[e^{rD}I^* + \Omega(\bar{X})]/N$ the corresponding DAC.

Proposition 2 *The optimal deployment schedule is such that:*

(i) the duration D^* and the quantity of accumulated cars $\bar{X}^*$ are independent of the CO_2 price, they satisfy the couple of equations:

$$\frac{\partial DAC(\bar{X}, D)}{\partial D} = rI^* + \frac{\partial I^*}{\partial D} = 0, \tag{13}$$

$$\frac{N}{r}\frac{\partial DAC(\bar{X}, D)}{\partial \bar{X}} = DAC(\bar{X}, D). \tag{14}$$

(ii) the optimal launching date T_l^* satisfies

$$p^{CO2}_{T_l^*} = e^{-rD^*} DAC(\bar{X}^*, D^*). \tag{15}$$

The proof is in Appendix C. This proposition calls for some comments. A decrease of the present CO_2 price p_0 only affects the launching date. This comes directly from the dynamics of the CO_2 price: Let us consider a change from p_0 to p'_0 with $p'_0 < p_0$ so that the whole trajectory of CO_2 prices is shifted. At a date T the new CO_2 price reaches the level of the old price at date 0 ($p'_0 e^{rT} = p_0$), the problem of the social planner is then simply shifted across time and so does its solution. The optimal trajectory is then simply postponed by T. Note that this reasoning holds with a CO_2 price that grows at any constant rate, possibly different from the discount rate.

The optimal duration D^* minimizes the DAC, and thus makes a trade-off, described by equation (13), between the cost of starting early (rI^*) and the benefits of spreading the effort across time ($\partial I^*/\partial D < 0$). This result is true even for a suboptimal accumulated quantity $\bar{X}$, as long as the ending date is fixed.[6] Furthermore, the condition (13) implies that the quantity of green cars at the beginning of the deployment should be null (cf Appendix C).

The optimal quantity of accumulated clean units satisfies equation (14) it does not minimize the DAC because of the environmental benefit associated to abatement during deployment. The impact of $\bar{X}$ on the total discounted production cost should be equalized with the CO_2 price at the end of deployment which is itself equal to the DAC from equation (9).

[6] The launching date and not the ending date should adjust to the change of duration reflecting that the DAC is a cost computed at the end of deployment.

If the ending date is suboptimal the optimal quantity of accumulated car should be adjusted accordingly.

Equation (14) reminds the classical result of supply theory that the minimum efficiency scale, which minimizes the average cost, equalizes marginal cost and average cost. At the optimal organization of production individual production should be equal to the minimum efficient scale, and total production is such that the price is equal to the minimum average cost (this latter equal to the marginal cost). The parallel with the present problem is not straightforward because of the inter-temporal dimension of our problem, but the choice of the launching date would correspond to the choice of the total production and the choice of X to the choice of individual production.

The parallel can be formalized as follow, introducing the total quantity of emissions $E = NT_e - \bar{X}$, and write the total discounted cost:

$$\tilde{\Gamma} = \frac{c_0 N}{r} + p_0 E + e^{-\frac{r}{N}(E+\bar{X})} \frac{N}{r} DAC(\bar{X}, D, \xi).$$

The cost to end the yearly emissions flow is $NDAC/r$. The choice of the total quantity of emissions makes a similar trade-off than the choice of the ending date, represented by eq. (9): a larger total quantity of emissions allows to bear this cost at later date. However, with a fixed quantity of emissions, the consequences of a change of $\bar{X}$ no longer involves the CO2 price: it has a direct effect on the deployment cost ($\partial DAC/\partial \bar{X}$), and also allows to postpone the spending by $1/N$ years.

Concerning the evaluation of a change of a deployment trajectory, possibly sub-optimal. It is not sufficient to only consider the effect on the DAC, the change of the quantity of accumulated car also matters. The following corollary states how both should be compared.

Corollary 2 *A change of a deployment schedule that increases the accumulated green units by* $\Delta\bar{X}$ *and the DAC by* ΔDAC *reduces the total discounted cost of the transition if and only if*

$$\frac{\Delta DAC}{DAC} < \frac{\Delta \bar{X}}{N/r}. \tag{16}$$

Proof. Assuming an optimal launching date, the effect of a small change is (from equations (11) and (10)):

$$\Delta\Gamma = e^{-rT_e}\left[\frac{N}{r}\Delta DAC - DAC\Delta\bar{X}\right] = e^{-rT_e}\frac{N}{r}DAC\left[\frac{\Delta DAC}{DAC} - \frac{\Delta\bar{X}}{N/r}\right].$$

■

Any change that reduces the DAC while maintaining the quantity $\bar{X}$ fixed is beneficial. Otherwise, the rate of change of the DAC should be compared with the ratio of the change of accumulated abatement during the transition to the discounted flow of abatement N/r.

2.4 The impact of the CO2 price growth rate

The model allows to understand the limits of changing perspective (from a single car to the whole technological deployment) and of the parallel between a deployment schedule and a huge clean plant. In this section we analyze the role of the CO_2 price growth rate on the launching date of a deployment schedule, possibly suboptimal.

The DAC, defined in Proposition 1, is theoretically valid only if the CO_2 price grows at the interest rate, which allows to neglect interim abatement. Assuming that the CO_2 price grows at the interest rate considerably simplifies the analysis of dynamic trajectory, by reducing to a single number p_0 the description of the whole CO_2 price path. Indeed, this helps getting a single number to characterize a time profile of abatement. With a different dynamic of the CO_2 price, interim abatements interact with the dynamic of the CO_2 price and impact the launching date. This effect is detailed in the following corollary.

Corollary 3 *If the CO_2 price grows at the rate $\rho \neq r$, $p_t^{CO2} = p_0 e^{\rho t}$, the optimal launching date of a deployment schedule $(\bar{X}, D, \xi)$ is such that:*

$$p^{CO2}_{T_l^*+D} = \frac{rIe^{rD} + (r\Omega(\bar{X}) - c_0 N)}{N - (\rho - r)\int_0^D e^{-(\rho-r)(D-\tau)}\xi_\tau d\tau}. \quad (17)$$

The proof is in Appendix D. If the CO_2 price does not grow at the interest rate, emissions have different present costs depending on the date at which they are emitted which explain that production during deployment should influence the launching date.

Insert Figure 1

Figure 1: The CO_2 price growth rate impact on the launching and ending dates of deployment and the corresponding CO_2 prices, with $\rho > r$.

If production during deployment is negligible, $\xi \simeq 0$, the CO_2 price at the end of the deployment, described by equation (17), is equal to the DAC, and is not affected by the growth rate of the CO_2 price. The launching date is then obtained by discounting the DAC with the CO_2 price growth rate ρ and not with the discount rate r which shows that the discounting of DAC in the formula (10) is related to the dynamics of the CO_2 price.

With non negligible interim abatement, the comparison between the CO_2 price growth rate and the interest rate determines whether later emissions are more costly than earlier emissions and the influence of interim abatement on the launching and ending dates. The CO_2 price at the ending date is described by equation (17); the DAC should be corrected by subtracting a term to the yearly abatement N in the denominator. The sign of this term is determined by the comparison between the growth rate of the CO_2 price and the interest rate, it reflects the gain (if $\rho > r$), or cost (if $\rho < r$), to postpone interim abatement.[7]

Figure 1 illustrates the Corollary with $\rho > r$. If the growth rate of the CO_2 price is larger than the interest rate. The ending CO_2 price is larger than in case of equality, given that delaying interim abatement reduces costs. However, the launching CO_2 price is lower than in the case of equality (cf Appendix D) because of the overall higher emission costs. More precisely, in Appendix D, we show that an increase of the CO_2 price growth rate both reduces the launching CO_2 price and increases the ending CO_2 price.

[7]Theoretically, both cases are possible: If the objective is to stabilize the concentration of CO_2 in the atmosphere, the CO_2 price should grow at a higher rate than the interest rate (e.g. Goulder and Mathai, 2000). To slow the extraction of fossill exhaustible resource the growth rate of the carbon price should be lower than the intereste rate, with constant extraction costs (e.g. Grimaud and Rouge, 2008).

3 Application to the case of Hydrogen versus Gasoline cars

3.1 The proposed trajectory and the associated static abatement costs

The application concerns the substitution of gasoline vehicles (Internal Combustion Engine vehicles) by hydrogen vehicles (Fuel Cell Electric vehicles). Our application focuses on Germany, a country in which some significant moves have already been made in favor of hydrogen. We built an Excel model based mostly on the data described in McKinsey & Company (2010). An exogenous trajectory from 2015 to 2050 generates cost dynamics involving three main components: manufacturing costs, fuel costs and infrastructure costs in constant € 2015 excluding value added tax (VAT). We briefly review the construction of our trajectory and its associated cost function as summarized in Table 1.[8]

The total passenger car fleet in Germany is assumed to increase from today's level of 47 million vehicles to 49.5 million in 2030. It is assumed to be stable from 2030 to 2050. Our exogenous deployment trajectory assumes a very progressive ramp up starting in 2015 up to a targeted market share of 15% in 2050, that is 7.5 million units for the hydrogen car park. It is noteworthy that there is no uncertainty surrounding that long-term result, once the deployment schedule is launched it will be sustained and will reach that target.

Hydrogen and gasoline cars are expected to have a ten year life time. The actual yearly production of hydrogen cars takes into account the increase in market share and the renewal of the car park. Based on this schedule one derives the cumulative production and construct a unit manufacturing cost for a hydrogen vehicle using a learning by doing component. We calibrate this component to approximate the actual costs obtained from McKinsey & Company (2010). This gives a decline of 6.8% for each doubling of cumulative production, and 62% from 2015 to 2050. The decline in the gasoline vehicle cost is assumed to be much lower.

[8] For a complete discussion of our cost assumptions the reader is referred to Creti et al. (2015). The Excel model is available from the authors upon request.

Fuel costs are derived from the fuel prices, the energy efficiency and the yearly kilometers, assumed to be 15 000 km per year. Fuel price for H2 comes from a mix of technologies (development and capital expenditures of energy producers are integrated in this cost). Three new carbon free technologies are progressively introduced and substituted to steam methane reforming: electrolysis based on renewable energies, bio-gas and carbon sequestration and capture. The fuel cost for hydrogen and gasoline includes the delivery cost to the refueling station. For gasoline prices we assume that the excise tax on imported petroleum is included since it represents an opportunity cost for importing oil. Note that the excise tax is in absolute value so its percentage in the gasoline price declines over time. The untaxed gasoline price (excluding excise and value added taxes) follows the oil price in the world market, assumed to increase at a constant rate of 1.4%. This 1.4% annual growth rate is consistent with IEA long term projections. The average gasoline market price per liter in Germany in 2014 was equal to 1,3 € (excluding value added tax).

The cost of infrastructure is derived from the number of hydrogen refueling stations which is derived from the required network to deliver the total hydrogen consumption at every time period. This cost includes the capital and operating costs of a station.

The CO_2 emissions for each type of vehicle are also given. These emissions depend on the respective energy efficiency and the CO_2 content of each fuel. The latter is declining over time for H2 production thanks to the progressive introduction of carbon-free technologies while the former is improving over time for gasoline vehicle.

	Unit	2015	2020	2025	2030	2050
Market size (car life time: 10 years, 15 000km/yr)	car park in k unit	1	95	453	1350	7500
Vehicle manufacturing cost						
Hydrogen car (maintenance 9.5% not incl.)	k€	60.0	37.7	32.1	28.6	22.8
Gasoline car (maintenance 12% not incl.)	k€	22.0	21.4	21.0	20.6	18.0
Relative price Hydrogen vs Gasoline	%	173%	77%	53%	39%	27%
Fuel costs						
Hydrogen production cost (delivery cost to station included)	€ /kg	7.0	5.8	6.1	6.3	6.8
Hydrogen consumption per 100 km	kg/100km	0.95	0.87	0.84	0.80	0.70
Gasoline price	€/ l	1.30	1.35	1.40	1.46	1.71
of which excise tax	%	50%	48%	47%	45%	38%
Gasoline consumption per 100 km	l/100km	7.04	6.8	6.5	6.3	5.8
Infrastructure costs						
Number of Hydrogen Refueling Stations	#	40	220	926	2234	9257
Capital cost per car (maintenance 5% not incl.)	k€	62.24	2.39	2.02	1.65	1.18
CO2 emissions						
Hydrogen	gCO_2/km	90	62	50	38	17
Gasoline	gCO_2/km	198	190	183	176	162

Table 1: Simplified Data Sheet

Table 2 gives the total cost of ownership (TCO) and its components for each technology for 2020, 2030 and 2050.[9] By construction the TCOs approximately converge in year 2050, the year at which the deployment is completed. The TCOs may also be compared with the figures obtained in Rösler et al. (2014), this is done in Table 3. The order of magnitude is similar, our figures are somewhat higher for 2020 but the difference decreases over time.

[9] Each cost component is obtained via Table 1 by adding the annualized capital cost, using the discount rate and the life time for the annualized factor, and the maintenance cost, using the capital cost and the percentage for maintenance. The Excel model is available from the authors upon request.

Prior to 2042 the static abatement cost is positive, then it becomes negative. The very high value obtained for 2020 is far above any reasonable assumption for a social cost of carbon at that date (1 433 €/tCO_2 in our analysis and 1 158 €/tCO_2 based on the TCO of Rösler et al. (2014)). This would strongly suggest that the cost for substituting hydrogen to gasoline in 2020 is far above its benefit in terms of avoided emissions. Could it still be economically reasonable to launch the proposed hydrogen schedule as early as 2015 so as to benefit from the avoided emissions in later years at a low cost, thanks to the learning-by-doing embedded in the manufacturing cost of hydrogen cars?

Year		2020	2030	2050
Vehicle cost (manufacturing and maintenance)				
Hydrogen		8	6.1	4.9
Gasoline		5.1	4.9	4.3
Fuel cost				
Hydrogen		0.8	0.8	0.7
Gasoline		1.4	1.4	1.5
Infrastructure cost for Hydrogen		0.4	0.2	0.2
Total Cost of Ownership				
TCO hydrogen		9.2	7.1	5.8
TCO gasoline		6.5	6.3	5.8
Delta TCO per vehicle		2.8	0.8	-.01
CO_2 emissions avoided	tCO_2/year	1.93	2.08	2.18
Abatement cost static approach	€ /tCO_2	1 433	399	-3.6

Table 2: Cost benefit analysis in k€ / year per vehicle

	2020		2040	
	Gasoline	Hydrogen	Gasoline	Hydrogen
Rösler et al. (2014)	5.9	7.8	5.7	5.7
authors	6.5	9.2	6.0	6.2

Table 3: Comparison of the Total Costs of Ownership in k€ / year per vehicle

3.2 The dynamic abatement cost for the hydrogen deployment schedule

We want to determine the DAC of the deployment schedule associated with the proposed trajectory that is making the transition over a given number of years at the rate defined by the proposed trajectory. From the DAC we shall determine the optimal launching time of this sub-optimal schedule. Recall the result of Proposition 1:

$$DAC = \frac{rI}{N}e^{rD} + \frac{r\Omega(\bar{X}) - c_0 N}{N}. \tag{18}$$

Our illustration does not directly fit for applying this result. Firstly, the life duration of each vehicle is not one year but ten years, this generates a distinction between the hydrogen car park and the annual production but this is taken into account in our calculation of the annual cost. Secondly, the TCO and the CO_2 emissions for gasoline vehicles are not constant over time but slightly decreasing. This means that shifting the deployment schedule over time is not neutral with respect to the cost and emissions of the gasoline vehicles. But the differences are small and we shall neglect this impact. Thirdly in our calculation the TCOs almost converge in 2050 and no detailed assumptions are made after this date. We shall approximate the terminal value in the right hand side of equation (14) assuming that TCOs remain constant afterward. Since LBD is certainly higher in FCEV than in ICE this will lead to an over-estimation of the abatement cost.

Giving due considerations to these caveats Proposition 1 provides a first order estimate. To compute this estimate we proceed as follows

- Time is discrete and not continuous, r stands for the yearly social discount rate; we use 4 % as the social discount rate;
- The duration of the transition is $D = T_e - T_l = 35$ and $e^{rD} = 1.04^{35} = 3.94$;
- As discussed above we assume that $\left[\frac{r\Omega}{N} - c_0\right] = \left[\text{TCO}_{H2} - \text{TCO}_{gas}\right]/a$ in which a is the difference in emissions per unit of car at the end of deployment. This gives -10 € /2.18 t/year= -3.57 €/tCO_2.

- In the analytical model N stands for total emissions of the dirty fleet, in the application it is the targeted hydrogen car park N_{car} times the difference in emissions per unit of car at the end of deployment. This gives $7.5M \times 2.18$t/year= 16.3Mt.

- The discounted cash flow for the deployment schedule, I, is obtained from our Excel model. We have $I = 21.6$ bn€.

Altogether this gives the following result $DAC = 21.6 \times 0.04 \times 3.94/16.3 - 3.57 = 205$ €/t. Note that we may alternatively compute the first component of the DAC as the yearly equivalent of the investment cost per unit of the final car park that is deployed divided by the CO_2 avoided per vehicle at that time.

To obtain the optimal launching time we discount the DAC: $205/1.04^{35} = 52$ €/t. This value is within most estimates of the social cost of carbon in 2015 (Tol, 2014) so that it does not seem uneconomical to launch this sub-optimal schedule at that date.[10]

The difference in reasoning between the dynamic and the static approaches may be contrasted as follows. In a static approach, the TCO of hydrogen is decreasing so the static abatement cost decreases as well, which pervades any analysis in terms of optimal time for deployment. In a dynamic approach, a whole deployment schedule is considered. It leads to the permanent substitution of a given car park of 7.5 M units of gasoline cars by hydrogen cars over a period of 35 years, from 2015 to 2050. Using the results from Table 1, we firstly observe that thanks to learning-by-doing the manufacturing cost of hydrogen cars will decrease from 60 k€ in 2015 to 22.8 k€ in 2050. Over time the infrastructure cost will also decline (Table 2). In 2050 the TCOs of hydrogen and gasoline cars will both converge to 5.8 k€ (Table 2). This convergence is achieved through a deployment cost of 21.6 bn€, that is 2 882 € per vehicle in the car park or $2882 * .04 = 115$ € per vehicle per year. Over time the avoided emissions will increase and stabilize to 2.18 t/year (Table 2). Altogether this gives

[10] Estimates depend on various assumptions most notably the pure time discount rate and the convexity of the damage function. For instance Nordhaus (2011) computes 12$ /tCO_2 (in 2005 US$) and Stern (2007) 68 $/t$CO_2$ while Golosov et al. (2014) find 15.5 $ and 135$ with their respective discount rate. At the country level, for France to reach its current policy objective, Quinet (2009) and Quinet (2013) suggest around 30 €/t for 2015; for the US, the EPA provides estimates (see https://www.epa.gov/climatechange/social-cost-carbon) the average of which is 56$ in 2015 with a 3% discount rate.

$115/2.18 = 53$ € /tCO_2 for the discounted dynamic abatement cost (neglecting the impact of terminal value which is $-3.57/1.04^{35} = -.9$ € /tCO_2).

3.3 Sensitivity Analysis and Discussion

We first show that the proposed schedule, keeping constant the total quantity accumulated and duration, does not minimize the investment cost. To do this we transfer a positive or negative amount of new vehicles schedule from the latest years to earlier years within the period 2040-2050. Based on our Excel model we find that the investment cost would decrease with more cars being deployed earlier. This suggests that the proposed schedule is too slow.

Our Excel model does not allow to derive the optimal duration and its associated schedule. For one thing we have not formalized the convexity of the cost function. As in many applied models this feature is introduced by setting capacity constraints at the earlier stage of deployment. Still we can test whether a shorter duration associated with less accumulated quantity would be preferable. Consider a duration $D = 25$ years instead of 35 years to reach 7.5 M units at the end of the deployment. Assuming that the schedule is left unchanged over the first 15 years our Excel model gives that the accumulated quantity would be $\bar{X} = 8.9$ M units instead of 13.4 M units. The corresponding DAC would be 139 € instead of 205 €. Applying Corollary 2 we see that the new schedule should be preferable. Its discounted DAC is 62 € instead of 52 € which means that it should start approximately 4 years later than the initial schedule but that it would end approximately 6 years earlier. On the practical side these calculations show that the optimal schedule has not a strong impact on our numerical estimate of the optimal launching date.

The robustness of our result can also be tested through a standard sensitivity analysis. For instance we can make a local exploration in which some parameters of the proposed trajectory are marginally changed. More precisely we identify the change in each parameter that would be consistent with a given targeted discounted adjustment abatement cost say of 30 €/t versus 80 €/t in 2015, a reasonable range for launching the deployment schedule in 2015. Our sensitivity analysis concerns the three parameters which are the most important: the market size of FECV in the total car park in 2050, the growth rate for the gasoline price, the learning rate for manufacturing costs. The detailed results of this analysis are given

Table 4. This sensitivity analysis suggests that our estimate of 52 €/t is fairly robust.

Discounted DAC		Market share	Gasoline price	Learning rate
scenario	€ per tCO_2	% of total market	% annual growth rate	% decrease every doubling of cumulative production
high	80	12	0.7	6.6
base	52	15	1.4	6.8
low	30	18	0.7	6.9

Table 4: Evolution of the discounted dynamic abatement cost with respect to the main parameters

Overall our analysis suggests that the deployment of hydrogen vehicles in 2015 to substitute gasoline vehicle is economically founded, giving the learning by doing in costs and the future benefit in reducing emissions. However a number of caveats should be pointed out. A long and progressive deployment schedule is necessary to achieve the cost reduction. Indeed 2050 is far away in the future and many factors not taken into account here may comfort or alleviate our conclusion. It would greatly help to obtain a much faster cost reduction in the production of hydrogen vehicles and a much faster decarbonisation of hydrogen production at a reasonable cost. In the current context of low carbon market price the actual deployment depends on substantial public policies both for infrastructure and demand side subsidies.

We have also considered that hydrogen and battery electric vehicles compete on different market segments: long range large cars for the former and low medium range for the latter. One of the reasons for this is the current limited autonomy and the long refilling time for BEV. Technical breakthroughs in battery conception and production may invalidate this assumption. In this respect one may also need to look at the whole value chain from electricity production to consumption and take into consideration the indirect benefits of H2 versus battery in terms of electricity storage.

Finally one may also consider that the increased awareness about pollution problems in cities will strongly encourage the introduction of zero emission vehicles including motorbikes, buses, trucks, trains... and that this social pressure will generate a complete change in behavioral habits leading to a reshuffling in modes of transportation. To really assess the validity of our conclusion our cost benefit analysis should be embedded into a more prospective scenario.

4 Conclusion

The main contribution of the paper consists in designing a decomposition methodology to disentangle the choice of the production schedule from the choice of the launching date in the search of the optimal trajectory. This leads to two interesting results. Firstly we extend the standard static notion of abatement cost associated to the substitution of a dirty unit by a clean one at some point of time to a dynamic abatement cost in which the all deployment trajectory is globally considered from its launching date. Second this dynamic abatement cost is also meaningful for a second best trajectory, which is often the case in applications where trajectories are defined through industrial and social considerations outside the scope of the modeling exercise. These results provide a simple framework for policy guidance. This is illustrated through an analysis of a trajectory in which gasoline vehicle (a.k.a. Internal Combustion Engine) are progressively replaced by hydrogen vehicle (a.k.a. Fuel Cell Electric Vehicle).

It would be interesting to extend this approach in several directions. From a theoretical point of view the dependence of the launching date on the learning rate for the optimal trajectory is worth to clarify (possible generalization of Corollary 1). Our decomposition methodology relies on a number of stationary assumptions which may be revisited in search for possible theoretical extensions. Indeed in the FECV case we have assumed that the minor efficiency gains in gasoline vehicle and the time increase in gasoline fuel costs do not invalidate the derivation of a dynamic abatement cost; this would be worth further work. A more elaborate extension would consider the simultaneous deployment of alternative clean technologies such as battery electric vehicle and hydrogen vehicle to be substituted to gasoline vehicle. This may possibly involve the introduction of consumers' preferences in which the role of product differentiation could be analyzed. Furthermore, our approach is optimistic, in that we assume that once the deployment starts, it is sustained all along the trajectory.

To be more realistic, uncertainty about consumers tastes and competing technologies costs should be introduced.

Another interesting extension would be to consider the decentralization issue of the optimal trajectory to the various players (manufacturers, H2 producers, network operators). These players need operate under a positive profit constraint assumptions. We have assumed an exogenous normative CO_2 price. There is no guaranty that the transfer of external benefit to the players can be enough to accommodate the positive profit constraints. Defining more operational policy instruments could be examined such as imposing a minimal rate of clean cars in the portfolio of manufacturers. We think that some answers to these various questions could be obtained while preserving the simplicity of our approach.

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Appendix

A Proof of Lemma 1

Proof. To minimize the total cost (2), let us introduce λ_t the co-state variable associated to the relation (3), and θ_t and δ_t the Lagrange multipliers associated to the two constraints (4) on x_t: it is positive (θ_t) and smaller that the total fleet size (δ_t). The first order conditions (together with the complementarity slackness conditions) are:

$$C_x(X_t, x_t) - c_o = p_t^{CO2} + \lambda_t + \theta_t - \delta_t \tag{19}$$

$$\dot{\lambda}_t - r\lambda_t = C_X(X_t, x_t). \tag{20}$$

The main step of the proof consists in proving that x_t is increasing if $x_t \in (0, N)$. This condition ensures that once $x_t > 0$ the number of green cars cannot come back to zero, and that x_t does not move when $x_t = N$. If x_t is strictly positive ($\theta_t = 0$) and lower than the total car fleet ($\delta_t = 0$), equation (19) becomes $C_x(X_t, x_t) - c_o = p_t^{CO2} + \lambda_t$ and taking the time derivative:

$$\begin{aligned} C_{xX}\dot{X}_t + C_{xx}\dot{x}_t &= \dot{p}_t^{CO2} + \dot{\lambda}_t \\ C_{xX}x_t + C_{xx}\dot{x}_t &= \dot{p}_t^{CO2} + r\lambda_t + C_X \qquad \text{thanks to eq. 20} \\ C_{xx}\dot{x}_t &= \dot{p}_t^{CO2} + r\lambda_t + [C_X - C_{xX}x] \end{aligned}$$

The last term of the right hand side is positive because $C_X(X, x)$ is concave with respect to x and $C_X(X, 0) = 0$ (since $C(X, 0) = 0, \forall X$). Since C_{xx}, p_t^{CO2} and r are all positive, $\dot{x}$ is also positive so that x_t is increasing through time.

Then, since the CO_2 price increases exponentially, x_t cannot be always null along an optimal trajectory. Then either $x_0 = 0$ or $x_0 > 0$. In the latter case $T_s = 0$, whereas in the former case T_s is the inf of the dates at which $x_t > 0$.

The ending date is finite, $T_e < +\infty$: From the above proof, when x_t is positive its time derivative is bounded below by a positive number, so x_t necessarily reaches N in a finite time.

Finally, equation (5) is obtained by integrating equation (20), between t and $+\infty$ (and using the boundary conditions $\lim_{t\to+\infty} e^{-rt}\lambda_t = 0$):

$$\lambda_t = -\int_t^{+\infty} e^{-r(\tau-t)} C_X(X_\tau, x_\tau) d\tau \tag{21}$$

and injecting this expression into equation (19).

■

B Proof of Lemma 2

Proof. If $C_{xx} = 0$, given that $C(X, 0) = 0$, $C_X(X, x) = C_{Xx}(X, x)x$. Then, we resort by reductio ad absurdum assuming $T_s < T_e$. Between the two dates the equation (5) is satisfied and taking its derivative with respect to t gives:

$$\begin{aligned}
\dot{p}_t^{CO2} &= C_{Xx}\dot{X}_t + C_{xx}\dot{x}_t - \dot{\lambda}_t \\
&= C_{Xx}x_t + 0 - [r\lambda_t + C_X] && \text{using eq. (20)} \\
&= C_{Xx}x_t - C_X + r\left[\int_t^{+\infty} e^{-r(\tau-t)} C_X(X_\tau, x_\tau) d\tau\right] && \text{from (21)}
\end{aligned}$$

Therefore, using that $C_X(X, x) = C_{Xx}(X, x)x$ in that case,

$$0 < \dot{p}_t^{CO2} = r\left[\int_t^{+\infty} e^{-r(\tau-t)} C_X(X_\tau, x_\tau) d\tau\right] \leq 0$$

a contradiction.

Therefore, the number of green cars jumps from 0 to N at date $T_s = T_e$, and the total discounted cost Γ could be written as a function of the date T_s:

$$\Gamma = \int_0^{T_s} e^{-rt}\left[(p_t^{CO2} + c_o).N\right] dt + e^{-rT_s}\Omega(0)$$

Along the optimal trajectory, T_s should minimize this function. Taking the derivative with respect to T_s in the equation above and setting it equal to zero gives the equation (6). ■

C Proof of Lemma 3, Proposition 2

From Lemma 1, the optimal trajectory $(x_t^*)_{t\in[0,+\infty)}$ can be described as the launching of a deployment schedule. There is therefore no loss to minimize the cost over the set of trajectories defined as the launching (and ending) of a deployment schedule. The trajectory obtained from the optimal launching of the optimal deployment schedule coincides with the optimal trajectory described in Lemma 3.

C.1 Proof of Lemma 3

Proof. Using the decomposition of the total discounted cost Γ provided by equation (11), the schedule ξ only influences the DAC and no other component of the cost. So that the ξ that minimizes Γ corresponds to the ξ that minimizes the DAC. A deployment schedule $(\bar{X}, D, \xi)$ and its DAC are only defined for ξ such that $\int_0^D \xi_\tau d\tau = \bar{X}$. The optimal ξ, for a given $\bar{X}$ and D, is then the solution of the optimization program:

$$\min_{\xi_\tau} \int_0^D e^{-r\tau}[C(X_\tau, \xi_\tau) - c_0\xi_\tau]d\tau \tag{22}$$

$$\text{s.t. } \dot{X}_t = \xi_\tau; \; 0 \leq \xi_\tau \leq N \text{ and } X_D = \bar{X}.$$

This result holds for any $\bar{X}$, D and T_l. ■

It is interesting to write the equations satisfied by ξ in order to recover the optimality conditions (5) satisfied by the optimal trajectory. Write the Lagrangian:

$$\mathcal{L} = e^{-r\tau}\left[C(X_\tau, \xi_\tau) - c_o\xi_\tau\right] - \mu_\tau\xi_\tau - \theta_\tau\xi_\tau - \delta_\tau(N - \xi_\tau) - \alpha\xi_\tau$$

in which μ_τ is the co-state variable of $\dot{X} = \xi_\tau$; δ_t and θ_t are the Lagrange multiplier of $\xi_\tau \leq N$; and $\xi_\tau \geq 0$ respectively, and α is the Lagrange multiplier of the constraint $\int_0^D \xi_\tau d\tau = \bar{X}$. The optimal ξ satisfies the equations:

$$e^{-r\tau}\left[C_x(X_\tau, \xi_\tau) - c_0\right] + \delta_\tau - \theta_\tau = \mu_\tau + \alpha$$

$$\dot{\mu}_\tau = e^{-r\tau}C_X(X_\tau, \xi_\tau), \; \mu_D = 0$$

$$\delta_\tau(N - \xi_\tau) = 0, \; \theta_\tau\xi_\tau = 0$$

Then, ξ_τ is increasing (similar reasoning than for the proof of Lemma 1), and integrating the second equation gives

$$e^{-r\tau}[C_x(X_\tau, \xi_\tau) - c_0] + \delta_\tau - \theta_\tau = \alpha - \int_\tau^D e^{-rs}C_X(X_s, \xi_s)ds \tag{23}$$

Together with the optimality conditions satisfied by $\bar{X}^*$, D^* and T_l^*, to be studied below, these first order conditions will coincide with 5. However, even for suboptimal $\bar{X}$ and D the schedule ξ should satisfies these equations, which then gives the minimized deployment cost $I^*(\bar{X}, D)$ and the associated $DAC(\bar{X}, D)$. The derivative of the deployment cost are:

$$\frac{\partial I^*}{\partial \bar{X}} = \alpha = e^{-rD} C_x(\bar{X}, \xi_D) - c_0 + \delta_D \tag{24}$$

$$\begin{aligned}\frac{\partial I^*}{\partial D} &= e^{-rD}\big[C(\bar{X}, \xi_D) - c_0 \xi_D\big] - \alpha \xi_D \\ &= e^{-rD}\big[C(\bar{X}, \xi_D) - c_0 \xi_D\big] - \Big[e^{-rD}\big[C_x(\bar{X}, \xi_D) - c_0\big] + \delta_D\Big]\xi_D \end{aligned}\tag{25}$$

C.2 Proof of Proposition 2

There are two possible strategies to prove that $\bar{X}^*$ and D^* are independent of the CO2 price p_0. One is sketched in the main text. The other consists in looking at first order conditions and showing that the optimal duration and accumulated quantity satisfy a pair of equation independent from the CO_2 price.

Proof. From the expression (11) of the total discounted cost, the optimal D^*, $\bar{X}^*$ and launching date satisfy the equations :

$$\begin{aligned} rI^* + \frac{\partial I^*}{\partial D} &= 0 \\ e^{rD}\frac{\partial I^*}{\partial X} + \frac{\partial \Omega}{\partial X} &= p_0 e^{rT_e} \\ p_0 e^{rT_e} &= \frac{1}{N}\left[re^{rD} I^*(\bar{X}, D) + r\Omega(\bar{X}) - c_0 N\right] \text{ using eq. (10)} \end{aligned}\tag{26}$$

The first equation corresponds to equation (13), the third correspond to (15). And injecting the third into the second gives equation (14) satisfied by $\bar{X}^*$.

The two equations (13) and (14) are independent of the CO_2 price or the launching date, the couple $\bar{X}^*$ and D^* is therefore independent of p_0, and so is the optimal deployment schedule $(\xi_\tau^*)_{\tau \in [0,D]}$ associated to them. ■

In addition, it is interesting to see how these equations together with equations (23) give back the equations (5).

From equations (14) and (24):

$$\frac{\partial I^*}{\partial \bar{X}} = \alpha = p_0 e^{rT_e} - \frac{\partial \Omega}{\partial \bar{X}} = p_0 e^{rT_e} - \int_{T_e}^{+\infty} e^{r(t-T_e)} C_X(\bar{X} + tN, N) dt$$

so that for $\xi_\tau \in (0, N)$ the equation (23) is equivalent to equation (5) with $\xi_\tau = x_{T_s+\tau}$.

Furthermore, one can show that equation (14) leads to $\xi_D = N$ and that equation (13) leads to $\xi_0 = 0$. This relatively fastidious proof is available upon request.

D Growth rate of the CO2 price

Proof of Corollary 3.

Proof. The total discounted cost should be written:

$$\Gamma(T_l) = \int_0^{T_l+D} e^{-rt}(c_o + p_t^{CO2})N dt + e^{-rT_l} I + e^{-r(T_l+D)}\Omega(\bar{X}) \\ - p_0 \int_{T_l}^{T_l+D} e^{(\rho-r)t}\xi_{t-T_l} dt$$

which is similar to (8) except that the second line above, the value of interim abatement, replaces $p_0\bar{X}$. The derivative of the second line with respect to T_l is (after an integration by parts):

$$-p_0 \int_{T_l}^{T_l+D} (\rho - r)e^{(\rho-r)t}\xi_{t-T_l} dt = -(\rho - r)p_{T_l+D}^{CO2} e^{-r(T_l+D)} \int_0^D e^{(\rho-r)(t-D)}\xi_t dt$$

so that the derivative of the discounted cost with respect to T_l is

$$\frac{\partial \Gamma}{\partial T_l} = e^{-r(T_l+D)} \left[(c_o + p_{T_l+D}^{CO2})N - rIe^{rD} - r\Omega(\bar{X}) - p_{T_l+D}^{CO2}(\rho - r) \int_0^D e^{(\rho-r)(t-D)}\xi_t \right] dt$$

This derivative is equal to zero at the optimal launching date (the second order condition is satisfied) which gives equation (17). ■

The following result is proved:

Result The ending (resp. launching) CO_2 price of scenario $(\xi_\tau)_{\tau\in[0,D]}$ is increasing (resp. decreasing) with respect to ρ if $\xi_0 = 0$ and $\xi'_\tau > 0$.

Proof.

We denote ξ'_τ the derivative with respect to time of ξ_τ.

The ending CO_2 price is given by equation (17). The derivative of its denominator with respect to ρ is:

$$\begin{aligned}&\int_0^D ((\rho - r)(D-\tau) - 1)e^{-(\rho-r)(D-\tau)}\xi_\tau d\tau \\ &= \underbrace{\left[(D-\tau)e^{-(\rho-r)(D-\tau)}\xi_\tau\right]_0^D}_{=0} \underbrace{- \int_0^D (D-\tau)e^{-(\rho-r)(D-\tau)}\xi'_\tau d\tau}_{<0}\end{aligned}$$

so the ending CO_2 price is increasing with respect to ρ.

The launching CO_2 price is:

$$p_{T_l^*}^{CO2} = \frac{rI + (r\Omega(\bar{X}) - c_0 N)e^{-rD}}{Ne^{(\rho-r)D} - (\rho - r)\int_0^D e^{(\rho-r)\tau}\xi_\tau d\tau} \tag{27}$$

The derivative of the denominator with respect to ρ is

$$\begin{aligned}&DNe^{(\rho-r)D} - \left[\tau e^{(\rho-r)\tau}\xi\right]_0^D + \int_0^D \tau e^{(\rho-r)\tau}\xi'_\tau d\tau \\ =&D(N - \xi(D))e^{(\rho-r)D} + \int_0^D \tau e^{(\rho-r)\tau}\xi'_\tau d\tau > 0\end{aligned}$$

so, the launching CO_2 price is decreasing with respect to ρ. ■