Extending the limits of the abatement cost

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Abstract

The paper examines the relevant cost benefit framework for state agencies investigating the potential of local projects to mitigate climate change. While these projects are typically limited in time and space, continuation paths need to be introduced to capture the benefits provided by the project. We propose a new metric that incorporates into the analytical framework the dynamic interactions between the project and its continuation. The new metric is a non trivial extension of the standard cost of abatement, and determines when to launch a project. We further analyse how the metric should be adjusted to compare two competing projects. Two illustrations make clear the novelty of our approach: the choice of the optimal mix of technologies for the electricity sector and the comparison between competing green technologies for mobility.
1 Introduction

While the introduction of a carbon tax would in theory provide the most efficient tool to induce decentralized decisions to mitigate greenhouse gas emissions, in practice, a large array of policies are implemented at the sectoral level. Among these policies the selection of pilot projects plays a major role since it is expected that these projects will generate future external gains through cost reductions and spillovers. The objective of this paper is to extend the standard methodology conventionally used in such studies. It has been motivated by our in depth involvement in a number of case studies such as EAS-Hymob (Brunet and Ponssard, 2017), Zero Emission Valley (Teyssier d’Orfeuil, 2020) in France, H-Vision in the Netherlands and HyNet NW in the UK (Athias, 2020), Fuel Cell Electric Buses for European metropolitan areas (Meunier et al., 2019).

Our approach may be seen as an extension of two recent papers (Baker and Khatami, 2019; Gillingham and Stock, 2018). Both papers insist on introducing dynamics in using the abatement cost. Baker and Khatami (2019) define the notion of levelized cost of carbon (LCC) to reflect the time value of money and how the costs and benefits of reducing carbon emissions may change over time. This leaves open the external dynamics induced for future projects. Gillingham and Stock (2018) insist on the role of these external effects by reviewing in detail the impact of learning-by-doing in two case studies (solar panels and electric vehicles), but do not elaborate on operational tools to systematically integrate them. We propose an analytical framework in which the time value of money, discounting of costs and abatements, learning-by-doing, uncertainty... can be explicitly discussed.

To illustrate our point consider a typical analysis which relies on the abatement cost to compare various alternatives to decarbonize a pre-existing polluting activity. Take for instance the case of steel production as discussed in (Friedmann et al., 2020). The analysis selects a future point in time and a reference scenario, it estimates the levelized incremental cost and the avoided emissions of each alternative. From these estimates it computes the abatement cost of each alternative to derive the preferred option. However there are two dynamic issues which are missing: what is the trajectory underlying the cost estimates at the future point in time and what are the future effects of the selected alternative. Some studies do provide cost estimates at several points in time, but without making explicit the dynamics between these estimates. Our analytical framework allows for simultaneously addressing trajectory and cost evolution through a closed loop solution.

More precisely we start from a finite life time project, presumably a pilot project intended to decarbonize a pre-existing polluting activity. We extend this project through a continuation path which details what are the future costs and abatements, possibly affected by the pilot project. We define the appropriate concept to analyse an extended project namely, the concept of dynamic abatement cost to be substituted to the static abatement cost. The concept generalizes a notion introduced in an earlier case study on Fuel Cell Electric Vehicles to properly reflect learning-by-doing effects (Creti et al., 2018). We show how the new concept generates metrics for solving two questions: when to launch an extended project and how to select among competing extended projects. Our contribution is intended to contribute to the development of operational tools for firms and state agencies investigating the potential cost benefit of local projects.

The paper is organized as follows. In section 2 we define the concept of dynamic abate-
ment cost, and the metrics for addressing the two questions of when and how. For clarity this is done assuming that the social cost of carbon grows at the same rate as the social discount rate, an economic assumption associated with Hotelling’s rule. In section 3 we revisit this simplifying assumption. In section 4 we introduce uncertainty through the existence of a backstop technology. In section 5 our approach is applied to two illustrations. Section 6 concludes.

2 The dynamic cost of abatement

2.1 The standard analysis and the paradox of the horizon

We consider a benevolent social planner who has to decide whether or not to fund a project which allows to decarbonize a pre-identified polluting activity (denoted as business as usual, BAU). Time is continuous and denoted \( t \in \mathbb{R}^+ \). BAU emits \( E \) tCO₂ per year. The project spreads over \( T \) years and reduces emissions by \( a_t \) thanks to incremental costs \( c_t \). By construction \( a_T = E \): the activity is fully decarbonized at the end of the project. The discount rate is denoted \( i \) and the Social Cost of Carbon, denoted \( P_t \) at time \( t \geq 0 \), grows at a rate \( \gamma \): \( P_t = e^{\gamma t} P_0 \). For the time being we assume that \( \gamma = i \). In theory, with a carbon budget, \( \gamma \) and \( i \) should indeed be equal (Hotelling’s rule); we shall come back to this assumption later on.

The total net benefit of implementing the project at time zero is:

\[
B = \int_0^T e^{-it}[P_t a_t - c_t]dt = P_0 \int_0^T a_t dt - \int_0^T e^{-it}c_t dt
\]  

(1)

A standard lemma follows

**Lemma 1** The project should be implemented iff:

\[
\frac{\int_0^T e^{-it}c_t dt}{\int_0^T a_t dt} \leq P_0.
\]  

(2)

The left hand-side of the equation coincides with the formula (2) in Baker and Khatami (2019) for the levelized cost of carbon (LCC) in which \( \beta = 0 \) since \( i = \gamma \). It corresponds to the standard abatement cost (MAC) when \( T = 0 \), that is \( MAC = c_0/E \). Note that if \( T > 0 \) the integration of the intermediary abatements is made without discounting because of the growth of the carbon price which exactly balances the social discount rate. The comparison between the LCC and the present carbon price determines whether the project should be implemented. There are two interrelated implicit assumptions in the cost-benefit analysis that leads to the use of the LCC: First, the time horizon is fixed and given by the project. Second, the project can only be launched today or never. There are two interrelated troubling consequences: First, even if today carbon price is lower than the LCC it will eventually be larger which suggests that any project will be worth implementing one day or the other. Second, the choice of the horizon \( T \) is somehow arbitrary and extending that horizon can make any project worth implementing today, this leads to what we propose to call the “puzzle of the horizon”.
Indeed suppose that the life time of the project is long. As $T$ increases, the numerator of the LCC is certainly bounded (because of the discounting) while the denominator will grow toward infinity (no discounting) as abatements close to $a_T = E$ are added. Consequently the LCC will go to zero, and any project with a sufficiently long horizon should be implemented regardless of the present carbon price.

**Corollary 1 (Puzzle of the horizon)** Any project to decarbonize a pre-identified polluting activity will be worth implementing for any present carbon price if its abatements are strictly positive for a long enough duration.

This result suggests that the correct question is not “whether” but “when” to implement a given project. However, to formally answer that question one needs to re-examine the choice of the time horizon. Firstly, if we take for granted that we search for decarbonizing a pre-identified polluting activity, we should consider a finite life project only if there is a full decarbonization continuation all along the future. Formulating this continuation path should be included in the cost benefit analysis. For convenience, denote the original project and its continuation path “the extended project”. Second, the corollary tells us that the interesting question can be subdivided into two parts: when an extended project project should be launched and how to compare two extended projects. We address these two points in sequence.

### 2.2 Three simple ways to extend the time horizon of a project

For clarity, we now distinguish between the calendar time $t \in \mathbb{R}^+$, and the project time $\tau$ which is the time since the beginning of the project. Recall that the project spreads over $T$ years, so $\tau \in [0, T]$, and reduces emissions by $a_\tau$ thanks to incremental costs $c_\tau$ relative to BAU at time $\tau$ after the beginning of the project. Note that this assumes there are no external factors related to calendar time which affect neither the abatements nor the project costs. We shall come back to this assumption.

At the end of the project, we consider that the emissions $E$ are perpetually eliminated thanks to a discounted cost $\tilde{C}$, which is the discounted sum of yearly expenses needed to operate and maintain the clean capital built during the project. Let us consider three simple continuation paths:

**Replication:** a constant cost $\bar{c}$ per tCO$_2$ should be spent each year, the continuation discounted cost is then

$$\tilde{C} = \int_0^{+\infty} e^{-\mu \bar{c} E} dt = \frac{\bar{c} E}{i}.$$

**Replication with learning-by-doing:** the cost per tCO$_2$ decreases over time at a rate $\lambda$ so that

$$\tilde{C} = \int_0^{+\infty} e^{-\mu \bar{c} E} e^{-\lambda \bar{c} E} dt = \frac{\bar{c} E}{i + \lambda}.$$

**Replication with learning-by-doing and spillovers:** the project creates side benefits $B$ once completed because of learning externalities for similar polluting activities...
\[ \bar{C} = \frac{\bar{C}}{i + \lambda} E - \bar{B}. \]

The benefits \( \bar{B} \) depends on the existence of similar polluting sites the decarbonization of which would be eased by the project considered. On these other sites, similar projects could be implemented, and need also be evaluated. The analysis of the coordinated evaluation of a portfolio of closely related projects is beyond the scope of the present article and worth further research. The two illustrations in Section 5 refine the first two cases to analyse some empirical issues.

### 2.3 When to launch an extended project

For a project to be launched at time \( s \), the associated extended project involves three parts. It is BAU until \( t = s \). It coincides with the project from time \( s \) until time \( s + T \), with abatements \( a_t \) and incremental costs \( c_t \). A continuation path involving a full decarbonization, \( a_t = E \) for all \( t \geq T + s \) for a discounted cost \( \bar{C} \). The choice of \( s \) should minimize the discounted social cost noted \( \Gamma(s) \) such that:

\[
\begin{align*}
\Gamma(s) &= \int_0^s e^{-is} P_t E dt + \int_s^{s+T} e^{-it} [P_t(E - a_{t-s}) + c_{t-s}] dt + e^{-(s+T)C} \\
&= \int_0^{s+T} e^{-it} P_t E dt + \int_s^{s+T} e^{-it} [c_{t-s} - P_t a_{t-s}] dt + e^{-(s+T)C} \\
&\quad \text{the project}
\end{align*}
\]

For convenience define as \( I \) the discounted cost of the project

\[ I = \int_0^T e^{-it} c_t dt \]  

We now define a dynamic abatement cost to be denoted as DAC such as

\[ DAC = \frac{i}{E} [I + e^{-iT} \bar{C}] \]

The optimal time \( s^* \) for launching the project is given by the following proposition.

**Proposition 1** The SCC at the optimal launching date \( s^* \) is equal to the DAC, that is

\[ P_{s^*} = \frac{i}{E} [I + e^{-iT} \bar{C}] \]

**Proof.** By postponing the launching time by one unit of time one reduces the total discounted cost by \( i(e^{-is} I + e^{-i(s+T)} \bar{C}) \), add emissions \( E \), and shifts intermediary abatements \( (a_t) \). With a carbon price growing at the interest rate \( P_t = e^{is} P_0 \) emissions are worth the same independently of their date, and shifting intermediary abatements has no impact on the total cost. The gain from postponing cost should be compared with \( P_t E \), that is the cost of adding \( E \) emissions. \( \blacksquare \)
To fix ideas consider a project with $T = 5$, $a_\tau = 1$ and $c_\tau = 100$ for all $\tau \in [1, 5]$. Assume $i = .05$ so that:

$$I = \int_{0}^{T} e^{-u}a_{\tau}dt = 100(1 - e^{25})/.05 = 442.40$$

$$LCC = \frac{I}{\int_{0}^{T} a_{\tau}dt} = 442.40/5 = 88.48$$

The DAC for $\lambda = 0$ is $\bar{c}/E = 100$. It is higher than the LCC of the project. Suppose now that we take $T = 20$, the LCC drops to 22.12 and it will go to zero as $T$ grows to infinity whereas the DAC, without learning is unaffected. The LCC would give the false impression that a long enough project should be launched at once while the DAC gives the correct answer to the optimal launching time of the extended project. Go back to $T = 5$ and suppose there is learning. With learning we have $DAC = i[I + e^{-iT}\bar{c}/(i + \lambda)]/E$. If $\lambda = .01$, the DAC is already lower than the LCC (it is worth 87.02). For $\lambda = .03$ it decreases to 70.79, a significant drop! Clearly our metric does not coincides with the LCC; it is specifically designed to answer the question of the optimal launching time of an extended project. And it depends on the formulation of the continuation path.

One can also look at the CO$_2$ price at the end of the project which is

$$P_{s+T} = DAC e^{iT} = i[Ie^{iT} + \bar{C}]/E$$

The cost of the project and its continuation are evaluated at the end of the project and not its beginning. The Social Cost of Carbon at time $s + T$ should be equal to the cost of decarbonizing a perpetual flow $E$ of emissions, intermediary abatements are irrelevant.

### 2.4 How to compare two extended projects

Consider two extended projects which decarbonize a pre-existing activity. Index by $k = 1, 2$ the characteristics of each project: $T_k$, $a_{\tau,k}$, $c_{\tau,k}$ and $\bar{C}_k$. Denote $\bar{s}_k$ the optimal launching time for the extended project $k$. The question is which of the two extended projects to select. The following proposition gives the answer. Let $A_k = \int_{0}^{T_k} a_{\tau,k}d\tau$ and $E_k = (\bar{s}_k + T_k)E - A_k$, $\bar{E}_k$ is the total emissions of the optimal extended project $k$. Comparing these emissions is enough to make the selection.

**Proposition 2** Between two competing projects 1 and 2 to decarbonize $E$, project 1 should be selected over project 2 if and only if it is associated with less total emissions computed with the optimal launching dates, that is $\bar{E}_1 < \bar{E}_2$.

**Proof.** By definition of the DAC, $P_\tau = DAC = i[I + e^{-iT}\bar{c}]/E$. For $\bar{s}_k$ to be optimal it must be that we have $P_0 = e^{-is_k}DAC_k$. It follows that comparing the social cost of the optimized extended project $\Gamma(\bar{s}_k)$ writes

$$\Gamma(\bar{s}_k) = P_0E(\bar{s}_k + T_k) - P_0A_k + e^{-is_k}DAC_k$$

Comparing the $\Gamma(\bar{s}_k)$ amounts to comparing the emissions from time 0 to $\bar{s}_k + T_k$, that is $\bar{E}_k$. ■

As a special case suppose that intermediary abatements are null, the following corollary clearly obtained. It nicely extends the standard MAC for ranking different projects.
Corollary 2 In the case of no intermediary abatements, the selected extended project is the one with the lower DAC and it should be launched at time $s_k$.

Note that in our metric the abatements are not discounted. This seems to contradict some standard applications in particular in the electricity sector and this point need be clarified. In the calculations of the levelized cost of energy (LCOE) produced quantities are discounted in order to compare the cost per MWh of technologies with different capital intensities and life spans, with the implicit assumption that these technologies could be used to produce similar load curves. Indeed, a well-known issue is the comparison between intermittent and dispatchable technologies (Joskow, 2011). Then, in some applications, the MAC is derived by putting at the denominator the discounted abatements over the life time of the project.

Let us explain how the issue of discounting relates to the comparison of the project with a hypothetical flexible one that costs $\tilde{c}$ per tCO$_2$ and would mimic the abatement profile $(a_t)$ followed by $E$. The cost of doing so with that flexible technology is

$$\int_0^T e^{-it} \tilde{c}a_t dt + \int_T^{+\infty} e^{-it} \tilde{c} E dt = \tilde{c} \left[ \int_0^T e^{-it} a_t dt + \int_T^{+\infty} e^{-it} E dt \right].$$

And the extended project under consideration is less costly than the flexible technology to abate the same profile if and only if

$$\tilde{c} > \frac{I + e^{-iT} \tilde{C}}{\int_0^T e^{-it} a_t dt + \int_T^{+\infty} e^{-it} E dt}.$$

This suggests that discounting abatements is the right approach when comparing competing projects. But our broader approach challenges that result, because the flexible technology should actually be used to decarbonize another profile that the project specific one; its flexibility is valuable and should be accounted for in the comparison. The flexible technology should be implemented to decarbonize the whole flow of emissions $E$ when the carbon price is equal to $\tilde{c}$ and the relevant comparison is given by Proposition 2 which can be restated as a comparison of discounted DACs.

Lemma 2 Proposition 2 may equivalently be rewritten as

$$DAC_1 e^{i\frac{T_1}{2} - \frac{i\Delta A}{2}} < DAC_2 e^{i\frac{T_2}{2} - \frac{i\Delta A}{2}}.$$

See Appendix A-1 for the proof. The following corollary follows.

Corollary 3 The project is preferable to a flexible technology with cost $\tilde{c}$ per tCO$_2$ if and only if

$$DAC e^{iT} e^{-i\frac{\Delta A}{2}} < \tilde{c}.$$

Proof. For the flexible technology we have $DAC_2 = \tilde{c}$; $T_2 = 0$ and $A_2 = 0$.

One way to interpret this result is to see that $A/E$ is the number of years $\tilde{T}$ such that total emissions $\tilde{TE}$ equals intermediary abatement $A$. Intermediary abatement $A$ save $A/E$ years of emissions, it is as if the project ends decarbonizing the flow $E$, at $T - A/E$ instead of $T$. The proper assessment consists then at computing the annualized cost at year $T - A/E$ and thus discount the $DAC$ by $i(T - A/E)$. 

7
3 What if the SCC does not grow as the social discount rate

Let us consider that the price of carbon grows at a rate \( \gamma \): \( P_t = e^{\gamma t} P_0 \), and denote \( \beta = i - \gamma \). This is the situation considered by Baker and Khatami (2019) who define the LCC as a function of \( \beta \):

\[
LCC(\beta) = \frac{\int_0^T e^{-\beta t} \alpha_t dt}{\int_0^T e^{-\beta t} \alpha_t dt}
\]

This LCC and its comparison with the current carbon price determines whether the benefits from abatements over the life time of the project are superior to its discounted cost. The same limitations applies, and we propose to choose the optimal launching date of the extended project by minimizing the overall social discounted cost \( \Gamma(s) \) given by equation (4).

**Proposition 3** The optimal launching date (if positive) is such that

\[
P_s = \frac{i[I + e^{-IT}C]}{\beta \int_0^T e^{-\beta t} \alpha_t dt + Ee^{-\beta T}}
\]

The proof of the proposition is in Appendix A-2. As before, the choice of the optimal launching date induces a trade-off between the benefit of postponing cost and the cost of postponing abatements. The right-hand side generalizes the DAC obtained for \( \beta = 0 \). With a carbon price not growing at the interest rate postponing intermediary abatements has an impact on the total discounted cost, it is costly if the carbon price grows slower than the interest rate (\( \beta > 0 \)) and beneficial otherwise. The denominator of the right-hand side is positive and decreasing with respect to \( \beta \) if the abatement \( \alpha_t \) are increasing, \(^1\) an increase of the growth rate of the carbon price advances the optimal launching time since it increases the value of following abatements.\(^2\)

Another way to obtain this result is to consider the carbon price at the end of the project:

\[
P_{s+T} = \frac{i[e^{IT}I + C]}{\beta \int_0^T e^{-\beta t} \alpha_t dt + E}
\]

The numerator of the right-hand side is the discounted cost at \( T \) of the project and its continuation, and the denominator is increasing with respect to \( \beta \) (if \( \alpha_t \) is increasing), so that an increase of growth rate of the carbon price decreases the carbon price at the end of the project.

Finally, it is interesting to note that the denominator in equation (8) is equal to \( \beta \) times the discounted sum of emissions \((\alpha_t)_{\tau \in [0,T]}\) followed by \( E \) after \( T \), discounted at rate \( \beta \), so that

\(^1\)The derivative of the denominator with respect to \( \beta \) is \( \int_0^T e^{-\beta t}(1 - \beta t) \alpha_t dt - Te^{-\beta T} \) which is, after an integration by part of the first term: \( -\int_0^T te^{-\beta t} \alpha_t dt \).

\(^2\)Indeed, in theory the project costs and abatements should be optimized according to the growth rate of the carbon price.
\[ P_s = \frac{i}{\beta} \int_0^T e^{-\beta t} a(t) dt + \int_T^{+\infty} e^{-\beta t} E dt \]

and the second fraction of the right-hand side could be interpreted as the LCC of the extended project, with the practical issue that the denominator is only well defined for \( \beta > 0 \) whereas \( \beta < 0 \) is well possible.

In France Quinet et al. (2019) gives the official reference price of CO\(_2\), so-called “valeur tutélaire du carbone”, which is growing at a non constant, decreasing, rate larger than the discount rate so \( \beta < 0 \) and increasing over time. Indeed, the valeur tutélaire du carbone was not determined as a welfare maximization under a carbon budget constraint that would have resulted in a Hotelling’s rule but as a carbon price that would ensure carbon neutrality in 2050, together with a carbon budget consistent with the Paris agreement and political constraints. As a consequence, according to our metric, this choice has delayed green investments.

4 Uncertainty and backstop technology

If a backstop technology may appear in the future to decarbonize the pre-existing polluting activity at no cost, the extended project should then be interrupted and the backstop implemented. If the backstop appears according to a Poisson process with rate \( \mu \), then at date \( t \) the backstop did not materialize with probability \( e^{-\mu t} \). Taking into account the possibility of the backstop into the computation of the social cost given by the expression (4) amounts to discounting by \( i + \mu \) instead of \( i \). Our analysis can then be reproduced replacing \( i \) with \( i + \mu \), so the cost \( I \) should be computed with \( i + \mu \) and the continuation cost \( \bar{C} \) also. For a continuation path such as replication with learning-by-doing it becomes:

\[ C(\mu) = \int_0^{+\infty} e^{-(i+\mu)t} e^{-\lambda t} E dt = \frac{\tilde{c}}{i + \lambda + \mu} E. \]

And Proposition 3 becomes

**Corollary 4** With a backstop technology characterized by a Poisson process with rate \( \mu \) the optimal launching date \( s \) is such that

\[ P_s = \frac{(i + \mu) \left[ \int_0^T e^{-(i+\mu)t} c(t) dt + e^{-(i+\mu)T} \bar{C}(\mu) \right]}{(\beta + \mu) \int_0^T e^{-(\beta+\mu)t} a(t) dt + E e^{-(\beta+\mu)T}}. \]  

(9)

An increase of the probability \( \mu \) decreases the denominator and has an ambiguous impact on the annualized cost at the numerator. If costs are decreasing (e.g. the project consists in a costly deployment of CAPEX before a cheaper OPEX costs) the numerator is increasing with respect to \( \mu \) and an increase of the probability of a backstop should delay the implementation of the project. A familiar result in the theory of option. Note also that intermediary abatements do intervene even if \( \beta = 0 \), that is so because the costs of the project are affected by the calendar time \( t \).
5 Illustrations

5.1 The optimal electricity mix of technologies in France at the 2050 horizon

Many empirical studies discuss the abatement cost based on LCOE calculations. This is especially the case in the electricity sector. Typically a reference year is selected, several portfolios of technologies are compared through their respective LCOE and potential emissions. Take one portfolio as the reference (business as usual, BAU). It is then a simple matter to derive the abatement cost of the portfolios relative to BAU.

In Corberand et al. (2021) such an analysis is carried on based on the methodology introduced in Friedmann et al. (2020) to compare various portfolios for the French electricity mix for a given level of consumption for year 2050. The main assumptions are the following: The demand level is given and fluctuates according to hour and day within the year. A portfolio is characterized by a mix of technologies. An optimization model is used to define the required capacities to face the fluctuating demand. The investment and operating costs of the corresponding capacities are defined, given their respective life time and assuming that they are greenfield; all costs are based on what is expected to be the state of the art of the technologies at year 2050.

For the sake of illustration consider the case in which BAU is 30% nuclear, 50% renewables, 10% hydro, and 10% natural gas while alternative portfolios would considerably reduce gas and related emissions. We focus on the so-called portfolio Proxy AMS which consists of 34% nuclear, 50% renewables, 10% hydro and the complement with methanation and methanisation. Note that the capacities in the two scenarios are different since the fluctuations of the demand is matched with a different mix of technologies. Electrolysers, batteries and gas turbines are used in Proxy AMS to cope with fluctuations. For a consumption level at 532 TWh, the yearly equivalent cost for BAU is estimated to be 45,815 M€ while the excess yearly equivalent cost for Proxy AMS is estimated to be 6,900 M€ which gives a LCOE at 86 €/MWh versus 99 €/MWh. The calculation of the avoided emissions amounts to 18.7 Mt CO₂, that is 0.35 t CO₂/MWh. This gives a LCC of 370 €/t CO₂.

As implemented to illustrate our approach would be to proceed as follows. Suppose that a smooth transition from BAU to Proxy AMS starts at some predefined year to be completed over ten years, \(T = 10\). Assume that the excess yearly cost would linearly converge from some multiple of 6,900 M€ to 6,900 M€. Say that the multiple is 2. The actual excess cost at year \(\tau\) is computed as this excess cost times the percentage of substitution at that year, for instance at year \(\tau = 1\), the percentage of implementation is 10% and \(c_1 = 6900 \times (1.+.9) \times .1 = 1311\), at year \(\tau = 2\) the percentage is 20% and we have \(c_2 = 6900 \times (1+.8) \times .2 = 2884\). At year \(\tau = 10\), the percentage of implementation is 100% and \(c_{10} = 6900 \times (1+0) \times 1 = 6900\). As for years after \(T\) let us assume that they remain constant (no learning-by-doing, no backstop technology).

As noted in Corollary 1 if one were to calculate the standard abatement cost for this extended Proxy AMS portfolio it would be zero since the discounted excess cost is finite while the total avoided emissions go to infinity (assuming a social cost of carbon that grows at the discount rate). The relevant question is at what time should one launch the substitution path? The answer suggested in this paper is to use the DAC. With our set of parameters.
using a discount rate of 3%, recalling that intermediary abatements are irrelevant, we get DAC=322 €/tCO₂. If the SCC is taken as 500 € tCO₂ in line with Quinet et al. (2019), the optimal launching time for Proxy AMS would be year 2035.

More realistic assumptions should be introduced such as having the set of costs for a portfolio depend on the calendar date accounting for exogenous technological change and a specific learning-by-doing for the portfolio. This exercise suggests some directions in which the LCOE analysis could be extended to compare portfolios.

5.2 The choice of battery versus hydrogen for mobility

The transportation sector needs to be decarbonized and two green technologies stand forward to achieve this goal: battery and fuel cell vehicles. The choice between these technologies depend on a number of factors such as range, refuelling time among others. The rate of learning-by-doing of each technology will play a significant role. To assess the evolution of the total cost of ownership (TCO) of each technology one needs to have an idea of the respective volumes and learning-by-doing. For instance consider the case of diesel (DB) buses the operator of which wants to clean either with battery (BEB) or hydrogen (FCEB) buses. In the short term FCEB are certainly more expensive than BEB but what about the long term? The long term TCO of FCB and BEB depend on the respective market shares that will be achieved which depend on the cost evolution which depend on the LBD rates...

We shall show that our metric provides an interesting starting point to formalize this issue as a closed loop optimization problem.

Start with a simple formalization of the situation. Time is continuous and denoted $t$. The discount rate is constant and denoted $i$. The social cost of carbon $P_0(t)$ grows at the rate $i$. There are $N$ DB to clean, the life time of a bus is one unit whatever the technology. The cost of DB is constant and normalized to zero. Its rate of emissions is also constant and denoted $E_D$. The cost of a green bus exhibits learning-by-doing (LBD), the form of which is assumed to be:

$$C(X_t, x_t) = [\xi + (\bar{c} - \xi)e^{-\lambda X_t}]x_t$$

in which, the variables $X_t$ and $x_t$ respectively stand for the cumulative production and the production at time $t$; the parameters $\xi$ and $\bar{c}$ respectively stands for the long and short term marginal costs; $\lambda$ represents LBD. These variables and parameters will be index by $B$ or $H$ to distinguish between BEB and FCEB.

Observe that the cost function is a special case of the one considered in Creti et al. (2018) in two respects. Firstly there is no convexity so that Lemma 2 in that paper applies: the transition to the green technology takes place instantaneously at a date $s$ to be determined either with BEB or FCEB. Secondly the precise form of LBD, which may be seen as an extension of the commonly used form in which $\xi = 0$, facilitates the calculation of the DAC. Indeed, once the transition takes place the future discounted cost denoted $\tilde{C}$ writes:

$$\tilde{C} = \int_0^{+\infty} e^{-it}[\xi + (\bar{c} - \xi)e^{-\lambda N}]N dt = \frac{1}{i}\xi N + (\bar{c} - \xi)\frac{N}{i + \lambda N}$$

so that the total social discounted cost $\Gamma(s)$, given by equation (4), simply writes:

$$\Gamma(s) = P_0s(NE_D) + e^{-is}\tilde{C},$$
which is minimized when $s$ solves

$$P_0(NE_D) = i e^{-is} \bar{C}$$

that is,

$$P_0 e^{is} = P_s = \frac{i \bar{C}}{NE_D} = \frac{1}{E_D} \left[ \bar{c} \left( \frac{i}{i + \lambda N} \right) - \frac{\lambda N}{i + \lambda N} \right]$$

The right hand-sides corresponds to the DAC. It may be seen as a weighted average between two static MACS: one with the short term marginal cost $\bar{c}$, and another with the long term one $\bar{c}$, the weights depending on the rate of LBD $\lambda$ and the size of the market $N$. The higher $\lambda$ and $N$, the lower the DAC which eventually converge to the standard MAC with the long term marginal cost. On the contrary, if $\lambda$ is null, the DAC coincides with the standard MAC with the short term marginal cost whatever the size of the market. These results are intuitive and well quantified with our metric. Coming back to the choice between BEB and FCEB, according to Corollary 2, the technology to be chosen is the one with the lowest DAC.

Three simple assumptions should be revisited to seriously address the choice between the two technologies. Firstly the non convexity of the cost function implies that the transition occurs simultaneously for the whole park of DB, a progressive transition would be more realistic. Secondly BEB and FCEB are assumed to be perfectly substitutable, a more elaborate market model should be introduced in which the competitive advantages of each technology be made explicit. Finally one should clarify the link between the LBD and the market: for instance can we expect some cost sharing between buses and trucks for some components? In spite of these limitations we believe that our preliminary analysis provides an interesting starting point.

6 Conclusion

The paper is intended to provide the relevant cost benefit framework for state agencies investigating the potential of local projects. Such projects typically explore different ways to decarbonize a pre-existing activity through the deployment of new technologies. As such they may be qualifi ed as pilot projects so that it is of major importance to incorporate into the analysis future cost reductions and externalities. We propose a new metric that incorporates the dynamics effects into the analytical framework. It is based on the notion of extended project, i.e. the project itself and a proposed continuation path in which decarbonization persists thanks to some incremental costs. We suggest different ways to define these incremental costs to capture the benefits provided by the project.

The new metric is a non trivial extension of the standard cost of abatement or levelized cost of carbon (Baker and Khatami, 2019). The latter ordinarily leads to what we call the puzzle of the horizon: any extended project which decarbonize a pre-existing activity would be good to launch immediately as long the social cost of carbon is increasing over time. Our metric allows answering the more relevant question which is when it would be appropriate to launch the extended project. Furthermore, we analyse how the metric should be adapted to compare two competing projects. The emphasis in the paper is mostly on methodology.
Much would remain to be done in any specific field study since a number of idiosyncratic features would appear. In some circumstances a local project would need to be enlarged through a life cycle analysis to meaningfully address the decarbonization issue. Sometimes a local project cannot be discussed independently of similar projects, the coordination of which having important consequences in terms of synergies and externalities. We provide some clues on how to proceed through two illustrations: the choice of the optimal mix of technologies for the electricity sector, a typical case in which standard abatement cost are used, and the comparison between competing green technologies for mobility, an important new topic for the energy transition.

A word of caution should be made to conclude: this paper does not address the question of abatement curves inferred from large multi-sector models through global studies made at the national or international level for the whole economy (note that such models have their own weaknesses, see Kesicki and Ekins, 2012). Local projects are seen as requiring a partial equilibrium framework while global ones would require a general equilibrium framework. While intrinsically simpler a number of issues remain open. Enlarging a local project study to encompass externalities such as suggested in this paper may be a more operational route than embedding the local project into a multi-sector model. We think that local projects are worth being studied and providing tools to analyze them is a relevant exercise.

References


### A Proof

#### A.1 Proof of Lemma 2

With a given project \( k = 1, 2 \) the optimal launching date \( \bar{s}_k \) is such that \( P_0e^{i\bar{s}_k} = DAC_k \) so

\[
\bar{s}_k = \frac{1}{i}[\ln(DAC_k) - \ln(P_0)]
\]

Recall that \( A_k = \int_0^{T_k} a_{r,k} d\tau \) stands for the total intermediary abatements of project for \( k = 1, 2 \).

Project 1 has less emissions \( E_1 \) than project 2 if and only if

\[
\bar{E}_1 < \bar{E}_2 \Leftrightarrow (\bar{s}_1 + T_1)E - A_1 < (\bar{s}_2 + T_2)E - A_2
\]

\[
\Leftrightarrow \left( \frac{1}{i}\ln(DAC_1) + T_1 \right)E - A_1 < \left( \frac{1}{i}\ln(DAC_2) + T_2 \right)E - A_2
\]

replacing \( \bar{s}_k \)

\[
\Leftrightarrow \ln(DAC_1) + iT_1 - i \frac{A_1}{E} < \ln(DAC_2) + iT_2 - i \frac{A_2}{E}
\]

multiplying by \( i/E \)

\[
\Leftrightarrow DAC_1e^{iT_1}e^{-i\frac{A_1}{E}} < DAC_2e^{iT_2}e^{-i\frac{A_2}{E}}
\]

#### A.2 Proof of Proposition 3

With \( P_t = P_0e^{\gamma t} \) the project abatements are worth (from today point of view):

\[
\int_s^{s+T} e^{-\beta t}P_0a_{t-s}dt = \int_s^{s+T} e^{-\beta t}P_0a_{t-s}dt = P_0e^{\beta s} \int_0^T e^{-\beta t}a_t dt.
\]

The total discounted cost (4) is then (using \( I \)):

\[
\Gamma(s) = \int_0^{s+T} e^{-\beta t}P_tEdt + e^{-is}[I + e^{-iT\bar{C}}] - P_0e^{-\beta s} \int_0^T e^{-\beta t}a_t dt
\]
Then the derivative with respect to the starting date $s$:

$$
\Gamma'(s) = e^{-i(s+T)}P_{s+T}E - ie^{-is}[I + e^{-iT}\bar{C}] + \beta P_0 e^{-\beta s}A(\beta) \\
= e^{-i(s+T)}P_s e^{\gamma T}E - ie^{-is}[I + e^{-iT}\bar{C}] + \beta P_0 e^{-\beta s}A(\beta)
$$

Which is then cancelled for $s$ that solves equation (8).