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Coordination of sectoral climate policies and life-cycle emissions

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Coordination of sectoral climate policies and life-cycle emissions

Quentin Hoarau and Guy Meunier*

Abstract

Drastically reducing greenhouse gas emissions involves numerous specific actions in each sector of the economy. The related costs and abatement potential of these measures are not independent from each other because of sectoral linkages. For instance, the carbon footprint of electric vehicles depends on the electricity mix, an issue that have received a large attention but little economic analysis. The present paper addresses the issue of sectoral policy coordination, especially when Pigovian carbon pricing is unavailable.

It analyzes the optimal allocation of mitigation effort among two vertically connected sectors, an upstream (e.g. electricity) and a downstream (e.g. transportation) one. The clean downstream technology consumes the upstream good and may thus shift emissions to the upstream sector. Using a simple partial equilibrium model, we characterize optimal second-best policies in presence of imperfect carbon taxation. We analyze how upstream emissions should be incorporated into the subsidy of the downstream technology, and consider the optimal coordination of both upstream and downstream subsidies to clean technologies.

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1 Introduction

The reduction of Greenhouse Gas (GHG) emissions requires a shift from fossil energy to low-carbon energy. For many energy uses (e.g. transport, industry, heating) such a shift may be achieved through electrification combined with low-carbon power sources (e.g. renewable, nuclear). Other decarbonization options involve a shift from fossil energy sources to hydrogen, which would also require low carbon production processes (from electrolysis and biomass). In these examples a downstream sector decarbonizes through technologies that consume an upstream good, the production of which also needs be decarbonized. As long as the upstream sector is not fully decarbonized, the decarbonization of downstream activities partly shifts emissions upstream. Life Cycle Assessments (LCA) have stressed such effect, notably for electric vehicles (e.g. Archsmith et al., 2015),[1] raising concerns about their carbon footprint. Allocating abatement effort across interconnected sectors is therefore an important policy issue. An usual method to rank mitigation options consists in the calculations of Marginal Abatement Cost (MAC), and their aggregation into MAC-curves (Gillingham and Stock, 2018). However, these MAC-curves have faced several criticisms by scholars, among which their inability to integrate sectoral interactions (Kesicki and Ekins, 2012).

This paper investigates the allocation of abatement efforts across a downstream and an upstream sector in such a configuration, and optimal policies when carbon pricing is constrained. It aims to clarify the relationship between life cycle emissions and optimal subsidies to low carbon technologies. We develop a partial equilibrium model with two sectors: an upstream and a downstream one. Households consume both goods. In each sector, a dirty and a clean technology are available. The downstream clean technology (e.g. electric vehicles) consumes a part of the upstream production (e.g. electricity) as an input. We analyze the optimal allocation of production for a given Social Cost of Carbon (SCC), and optimal policies in a flexible setting in which dirty goods may not be taxed at the Pigovian level.

Our contributions are threefold. First, we describe the market equilibrium and the influence of each instrument (tax and subsidy in each sector) on total emissions. Surprisingly, total emissions do not systematically decrease with subsidy on the downstream clean technology or tax on the upstream dirty technology. We identify, mutually exclusive, conditions under which emissions can increase with either the downstream subsidy or the upstream tax. Consequently, at the optimum, an increase of the SCC may involve an increase of upstream emissions because of the deployment of the clean downstream technology. This is more likely the less elastic upstream demand and clean technology supply are.

Second, we analyze how upstream emissions should be integrated into the computation of downstream Marginal Abatement Cost (MAC) and the subsidy of the clean downstream technology. At the optimum, there are multiple consistent ways to compute the MAC of the clean downstream technology. Upstream emissions (at the denominator) should be inte-

[1] See Hajjaji et al. (2013) for a LCA of hydrogen production. The case of hydrogen involves three stages: the fuel market (dowsntream), hydrogen production (upstream 1), and electricity production (upstream 2) in the case of electrolysis. Another more prospective example is cultured meat which may abate cattle emissions but requires a lot of energy that could be clean electricity (Tuomisto and Teixeira de Mattos, 2011; Mattick et al., 2015).

grated consistently with the cost considered (at the numerator). Furthermore, the optimal downstream subsidy should be corrected for upstream carbon mispricing and the subsidy to the clean upstream technology. The emissions associated with the marginal, and not the average, upstream unit matter and these emissions depends upon the regulatory instrument used in the upstream sector. We relate the formula obtained to the concept of Consequential Life Cycle Assessment (CLCA) which has been developed to overcome some limitations of standard "attributional" LCA by integrating adjustment of relevant quantities (Earles and Halog, 2011).

Third, we analyze the joint optimization of subsidies in both sectors. We show that the optimal upstream subsidy does not directly incorporate features of the downstream sector, whereas the downstream optimal subsidy depends upon the upstream sector characteristics, and notably the difference between the SCC and the upstream carbon tax. This asymmetry is due to the fact that the clean downstream indifferently consume both clean and dirty upstream production, while the clean upstream production can only be consumed by the clean and not the dirty downstream productions.

The present work bridges the gap between economic models of the energy transition and LCA approaches in industrial ecology. The latter raises questions for climate policy design that are not addressed in the former. Some of these questions are due to the lack of an exhaustive and efficient pricing of emissions observed (World Bank, 2021). With an exhaustive Pigovian tax, sectoral interactions and life cycle emissions do not need be considered in policy design. But the lack of carbon pricing, and the pervasiveness of subsidies, calls for analysis of second-best policies, and notably subsidies to clean technologies.

Our comparative static results are related to a literature that identifies mechanisms through which a subsidy to abatement could increases total emissions, through free entry (Baumol and Oates, 1975) or general equilibrium effects (Kohn, 1992; Mestelman, 1972, 1982). In the present work a different, and simpler, mechanism is identified, total emissions increase because of upstream polluting production. However, we also identify conditions under which a tax on a polluting (upstream) technology could increase total emissions because of downstream consequences.

From a normative perspective, in their seminal work Lipsey and Lancaster (1956) establish that optimality conditions guiding policy instruments should be modified to integrate pre-existing distortions. Indeed, a too small tax on a polluting good justifies subsidizing clean substitutes. The literature on carbon leakage has analyzed how the regulation of domestic emissions (via tax or tradable permits) should be complemented by subsidies (possibly via output based rebates) to domestic goods with unregulated foreign substitutes (Fischer and Fox, 2007, 2012; Meunier et al., 2017; Fowlie and Reguant, 2021). Notably Meunier et al. (2017) and Fowlie and Reguant (2021) analyze how the optimal subsidy depends on the sensitivity of foreign production to home production and foreign emission intensity. The fact that marginal and not average intensities matter for the optimal subsidy is also present in our analysis. Indeed, the domestic-foreign relationship differs from the downstream-upstream one, and these articles do not consider foreign regulations whereas we consider upstream regulation. Galinato and Yoder (2010) considers the optimal combination of tax and subsidy under a net-revenue constrained carbon tax and subsidy program, which explains a departure from the Pigovian rule. They do not model sectoral interconnections, even though they provide numerical illustrations for the electricity and transport sectors because they consider (second generation) biofuels as the clean transport technology and not electric cars. The combination of their analysis with our model is a path for future research.

A special case of carbon leakage related to our analysis arise with bio-energies (biofuels and wood energy) the carbon footprint of which depends upon life-cycle considerations. Emissions arising by their consumption are partly compensated by carbon off-takes at the production stage but several economic analysis have stressed that direct and indirect land use changes (mostly deforestation) can severely reduce their net climate footprints (e.g. Searchinger et al., 2008; Keeney and Hertel, 2009). Even though bio-energies do not perfectly fit our framework there are similarities,[2] and how to design support for bio-energies taking into account life cycle emissions is also a controversial and key policy question for the energetic transition. To our knowledge, the analysis of the coordination of bio-energy subsidies and upstream (land-use, agriculture) policies has not been done.[3]

The relationship between policies on electric mobility and electricity production has been investigated by Holland et al. (2015, 2021) and Gillingham et al. (2021). Holland et al. (2015) analyse optimal second-best subsidy to electric vehicles and how they should integrate emissions from power production. Holland et al. (2021) consider the transition of the transportation sector with an exogenously decarbonizing power sector. In both articles they do not consider the impact of electricity regulation on the optimal vehicle subsidy, and possibly the joint optimization of policies. Gillingham et al. (2021) analyze how the regulation of the power sector influences the environmental impacts of electric vehicles. They show that a carbon tax in the power sector, by making coal instead of gas the marginal power source, could deteriorate the environmental benefit of electric vehicles. This result on upstream carbon taxation differs from ours that rests on market interactions between the two sectors which are not modeled in their article. Furthermore, their analysis is descriptive and they do not analyze optimal policies.

To overcome some limitations of "attributional" LCA Consequential Life-Cycle Assessments (CLCA) integrate economic mechanisms in order to assess the consequences of a change in the quantity of the good under scrutiny or of the policy supporting it (Earles and Halog, 2011; Rajagopal, 2014). It has been notably used to discuss the carbon footprint of biofuel policies by integrating direct and indirect land-use changes (Rajagopal and Zilberman, 2013; Bento and Klotz, 2014).[4] However, these works give little recommendations on how to design adequate policies that would include sector interactions.

The article is organized as follow. The model is introduced in Section 2. In Section 3 we analyze the social optimum. Optimal policies are described in Section 4. Some numerical illustrations are discussed in section 5. Section 6 concludes.

[2]The parallel would be the following: the fuel market is the downstream sector with biofuel as the clean technology, and the agriculture and forestry sectors would be the upstream sector. Contrary to the situations we have in mind, there is no clean technology that might decarbonize the upstream sector, but several competing uses of land with different carbon footprints.

[3]Hoel and Sletten (2016); Hoel (2020) analyze first-best policies to take into account the dynamic of forest carbon sequestration. Tahvonen and Rautiainen (2017) analyze optimal forest management and second best policies, to limit the financial burden of subsidizing exhaustively forest carbon storage, but do not explicitly model the downstream fuel sector.

[4]See Ahlgren and Di Lucia (2014); De Cara et al. (2012) for reviews of the literature on biofuel climate impact which stress the diversity of modeling choices and notably the differences between economic and CLCA approaches.

2 The analytical framework

We consider a partial equilibrium model with two interrelated sectors, an upstream sector (e.g. electricity) and a downstream sector (e.g. road transport) labeled $i \in \{U, D\}$. Both goods are consumed by households, and both can be produced with two technologies: a "dirty" polluting technology and a "clean" emission-free technology labeled $j \in \{d, c\}$. The clean downstream technology uses the upstream good (electricity is both consumed by households and by electric cars). The structure of the model is shown on Figure 1.

For each sector $i = U, D$ the total quantity consumed by households is Q_i and the associated gross consumers surplus is $S_i(Q_i)$, with $S_i' > 0$, and $S_i'' < 0$. On the production side, in sector $i = U, D$ the total quantity produced is $q_{id} + q_{ic}$ the sum of dirty and clean productions, with production costs $C_{ij}(q_{ij})$ with $j = d, c$. Cost functions are positive, increasing and convex, $C_{ij}' > 0$ and $C_{ij}'' \geq 0$.[5] Each clean downstream unit consume θ units of the upstream good so that the total quantity produced $q_{Uc} + q_{Ud}$ is equal to the quantity consumed by households Q_u and by the downstream clean variety θq_{Dc}: $q_{Ud} + q_{Uc} = Q_U + \theta q_{Dc}$. We will refer to θ as the *linkage intensity*. In sector i, each unit produced by the dirty technology emits α_i tons of CO_2. We denote μ (in \$ per tCO_2) the Social Cost of Carbon (SCC). Total welfare is then:

$$W(\mathbf{q}, \mu) = \sum_i S_i(Q_i) - \sum_{ij} C_{ij}(q_{ij}) - \mu[\alpha_D q_{Dd} + \alpha_U q_{Ud}] \tag{1}$$

subject to $Q_D = q_{Dd} + q_{Dc}$ and $Q_U + \theta q_{Uc} = q_{Ud} + q_{Uc}$ and $q_{ij} \geq 0$ for $i = U, D$ and $j = d, c$.

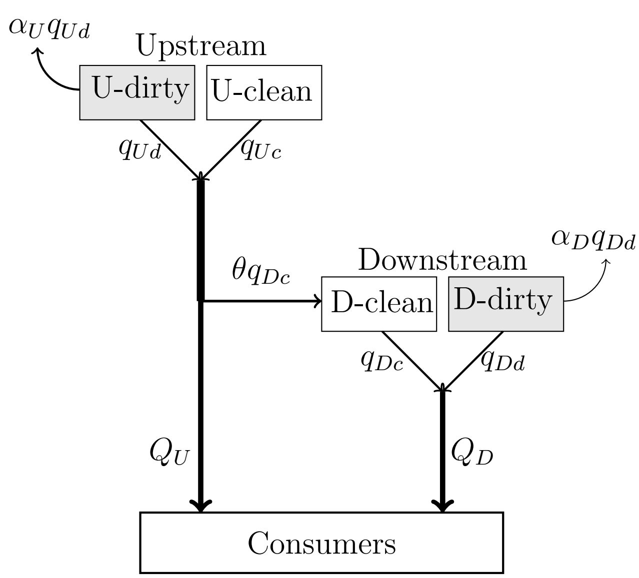

Figure 1: The structure of the model.

[5]For the case of electricity and transportation, the convexity of the clean technology costs is notably due to: For renewable energy, sites scarcity, storage and transportation costs. For electric cars, it mainly comes from the increasing cost associated with density (urban vs rural) and types (weight) of vehicles.

In sector $i = U, D$, the selling price of good i is p_i, there is a tax t_i on dirty units and a subsidy s_i on clean units, both can indeed be negative. Net consumer surplus is

$$CS_i(Q_i, p_i) = S_i(Q_i) - p_i Q_i,$$

and the inverse demand function is $P_i(Q_i) = S_i'(Q_i)$.[6] The profit of dirty and clean producers are

$$\pi_{id} = p_i . q_{id} - t_i q_{id} - C_{id}(q_{id}) \quad \text{for } i = U, D \tag{2a}$$

$$\pi_{Uc} = p_U q_{Uc} + s_U q_{Uc} - C_{Uc}(q_{Uc}) \tag{2b}$$

$$\pi_{Dc} = p_D q_{Dc} + s_D q_{Dc} - C_{Dc}(q_{Dc}) - p_U \theta q_{Dc}.. \tag{2c}$$

Both consumers an producers are assumed to be price takers, and respectively maximizes consumers net surplus and producers profit, prices clear both markets. Indeed, it is equivalent to consider that a single representative firm maximizes the aggregate profit over the two sectors or that a multitude of small producers are doing so for each sector and technology.

Sectoral and total welfare can then be rewritten as the sum of consumer net surplus, producer profit and tax revenues, possibly split between the two sectors:

$$W_i = CS_i + \pi_{id} + \pi_{ic} + t_i q_{id} - s_i q_{ic} - \mu \alpha_i q_{id} \quad \text{for } i = U, D \tag{3a}$$

$$W = W_U + W_D \tag{3b}$$

To analyse the adjustment of quantities to an increase of the SCC or regulatory instruments the following notation helps:

$$\Gamma_{ij} = \begin{cases} \frac{C_{ij}''(q_{ij})}{-S_i''(Q_i)} & \text{if } q_{ij} > 0 \\ +\infty & \text{if } q_{ij} = 0 \end{cases} \tag{4a}$$

$$\Lambda = \frac{S_U''}{S_D''} \tag{4b}$$

The parameter Γ_{ij} is the ratio between the slope of the supply curve of good ij relative and the slope of the price function, which represents the relative adjustment of the supply of good ij compared to the reduction of demand induced by an increase of the price.

Finally, we assume that with a small SCC only the dirty technology is used, and for a sufficiently large one there are positive quantities consumed in both sectors supplied with the clean technologies.

Assumption 1 *There are $Q_D^0 > 0$ and $Q_U^0 > 0$ such that*

$$S_i'(Q_i^0) = C_{id}'(Q_i^0) < C_{ic}'(0) \text{ for } i = U, D. \tag{5a}$$

And there are $Q_D^1 > 0$ and $Q_U^1 > 0$ such that

$$S_U'(Q_U^1) = C_{Uc}'(Q_U^1 + \theta Q_D^1) \text{ and } S_D'(Q_D^1) = C_{Dc}'(Q_D^1) + \theta C_{Uc}'(Q_U^1 + \theta Q_D^1) \tag{5b}$$

[6] Consumers are assumed to be price takers, and the demand function, $P_i^{-1}(p_i)$, maximizes the net consumer surplus.

The static framework can be used to analyze a dynamic transition along which the SCC increases and the economy moves from a state with only the dirty technology to a fully clean situation. Along that transition multiple technology mixes may arise as the clean technologies are progressively phased in and dirty technologies phased out.

The following comments have to be made on the previous modeling choices. First, we consider perfect substitutability on the consumption side between technologies in each sector. It simplifies the analysis and help focuses on the impacts of sectoral linkage, it also allows to have a well defined MAC of substituting a dirty unit by a clean one. In the power sector technologies are not perfect substitutes because of storage cost and the variability of demand and renewable supply (e.g. Baranes et al., 2017). We consider that the convexity of the upstream costs includes storage costs (as in Coulomb et al., 2019). In the mobility sector, electric and gasoline vehicles are more or less substitutable depending on the use (distance traveled per trip and per year, population density, weather conditions...). Second, the downstream clean technology cannot discriminate among upstream technologies, and consumes the same mix as other consumers.[7]

Third, our framework is static and does not include dynamic aspects such as adjustment cost, learning-by-doing, or sectoral inertia. Transition aspects will be appraised by comparative statics on the social cost of carbon. However, linear investment costs, stable over time, could be considered included in the production cost of clean technologies so that their progressive deployment would be associated with an investment.[8] Fourth, a peculiar type of sectoral linkage is considered here. The downstream clean technology creates a vertical sectoral linkage with the "upstream" sector. A more general and realistic setting would introduce sectoral relationships in an input/output framework, all sectors would be already linked before the introduction of the downstream clean technologies which would be associated with other technical coefficients.

To get explicit formula and make simulations, we will use the following quadratic specification (see Appendix A for the expressions of Q_i^0 and Q_i^1, $i = U, D$):

Specification 1 *For* $i \in \{U, D\}, j \in \{e, d\}$

$$S_i(Q_i) = a_i Q_i - \frac{b_i}{2} Q_i^2 \tag{6a}$$

$$C_{ij}(q_{ij}) = c_{ij} q_{ij} + \frac{b_i \Gamma_{ij}}{2} q_{ij}^2 \tag{6b}$$

with $a_i, b_i, c_{ij}, \Gamma_{ij}$ *all nonnegative real numbers.*

3 Optimal allocation

We first consider the optimal allocation and clarify the relationship between marginal abatement costs (MAC) and life cycle emissions. Indeed, MACs, obtained by substituting a dirty

[7] For instance, we do not consider the possibility to charge electric cars at night, so that the content of the electricity used to charge is not exactly the same as the total mix of the grid.

[8] For dirty technologies, the situation is less simple since they are phased out and some capacity may remain idle.

by a clean units, in both sectors should be equalized with the SCC. For a given SCC, the cost of the clean downstream technology depends upon the upstream sector consumers surplus and production costs. Upstream emissions should be encompassed in the computation of the downstream MAC consistently with the cost considered (Lemma 1).

The optimal allocation $\mathbf{q}^{FB}(\mu) = (q_{ij}^{FB}(\mu))_{i,j}$ maximizes welfare (1). Denoting ϕ_{ij} the Lagrange multiplier of the positivity constraint $q_{ij} \geq 0$, the first order conditions are:

$$\begin{aligned}
P_U(Q_U) &= C'_{Ud}(q_{Ud}) + \alpha_U \mu \ - \phi_{Ud} && \text{(7a)}\\
&= C'_{Uc}(q_{Uc}) - \phi_{Uc} && \text{(7b)}\\
P_D(Q_D) &= C'_{Dd}(q_{Dd}) + \alpha_D \mu - \phi_{Dd} && \text{(7c)}\\
&= C'_{Dc}(q_{Dc}) + \theta P_U(Q_U) - \phi_{Uc} && \text{(7d)}\\
Q_U + \theta q_{Dc} &= q_{Ud} + q_{Uc} && \text{(7e)}\\
Q_D &= q_{Dc} + q_{Dd} && \text{(7f)}
\end{aligned}$$

At the optimal allocation in each sector a positive quantity is produced and consumed thanks to Assumption 1, and the marginal consumer surplus is equal to the marginal costs of each technology used. Note that the marginal cost of the clean downstream technology encompasses the marginal benefit from the upstream good consumption P_U.

Lemma 1 *At the optimal allocation, if all technologies are used, the SCC is equal to the MACs of substituting a dirty by a clean unit in both sectors. In the downstream sector, a relevant MAC should weight similarly upstream costs and upstream emissions :*

$$\forall r \in [0,1], \ \mu = \frac{C'_{Dc} + \theta[(1-r)C'_{Uc} + rC'_{Ud}] - C'_{Dd}}{\alpha_D - \theta r \alpha_U} \tag{8}$$

In each sector, there are two ways to reduce emissions: reducing demand or substituting a dirty unit by a clean one. If all quantities are positive, at the optimal allocation the MAC associated with each option should be equal to the SCC. These MAC should be computed with direct emissions:

$$\mu = \frac{C'_{Uc} - C'_{Ud}}{\alpha_U} = \frac{C'_{Dc} + \theta P_U - C'_{Dd}}{\alpha_D}$$

At first glance, indirect emissions of the downstream clean technology do not intervene in those formula. However, the marginal cost of the downstream clean technology encompasses the marginal value of the upstream good P_U which depends upon the SCC, which is problematic if we want to compare MACs with the SCC. Replacing the expression of P_U with either equations (7a) or (7b) gives the two relations :

$$\mu = \frac{C'_{Dc} + \theta C'_{Uc} - C'_{Dd}}{\alpha_D} = \frac{C'_{Dc} + \theta C'_{Ud} - C'_{Dd}}{\alpha_D - \theta \alpha_U}$$

These equations tell us that the upstream emissions taken into account at the denominator of the MAC should be consistent with the upstream cost at the numerator. Indeed, it works with any weighting of the two technologies as long as marginal costs and emissions rates are similarly weighted.

It is possible that not all technologies are used. Indeed, for small (resp. large) SCC only dirty (resp. clean) technologies are used. In between, all configurations can arise depending on parameter values. Indeed, if a clean technology is not used then the MAC associated to it is below the SCC. Furthermore, the clean downstream quantity is not used if indirect upstream emissions are larger than downstream ones.

Lemma 2 *If $\alpha_D < \theta\alpha_U$, then the clean downstream quantity is null if the dirty upstream quantity is positive.*

The proof is straightforward and relies on Assumption 1:

$$C'_{Dc}(0) + \theta(C'_{Ud} + \alpha_U\mu) \geq C'_{Dd}(Q_D^{FB}) + \alpha_D\mu$$

Together with equations (7c) and (7b) implies that q_{Ud}^{FB} is positive only if q_{Dc}^{FB} is null. In that case, in a dynamic perspective, as the SCC increases the clean downstream technology is used only once the upstream sector is fully decarbonized. We assume that it is not the case for the rest of the article.

Assumption 2 *Downstream emission intensity α_D is larger than indirect dirty emission $\theta\alpha_U$.*

4 Optimal policies

In this section we analyze the policy implications of sectoral linkages. First, we begin by deriving the market equilibrium of the model. From this market equilibrium, we assess the impact of each policy instruments on total and sectoral emissions. Second, we derive the optimal downstream policy with fixed upstream policy instruments. Third, we derive the overall second-best instruments under imperfect carbon pricing either in both sectors, or only in one.

4.1 Market equilibrium, Pigovian taxation

Market equilibrium prices and quantities satisfy the equations, denoting ψ_{ij} the Lagrange multiplier of the positivity constraint $q_{ij} \geq 0$:

$$S'_i(Q_i) = p_i = C'_{id}(q_{id}) + t_i - \psi_{id} \text{ for } i = U, D \tag{9a}$$

$$P_D(Q_D) = C'_{Dc}(q_{Dc}) + \theta p_U - s_D + \psi_{Dc} \tag{9b}$$

$$P_U(Q_U) = C'_{Uc}(q_{Uc}) - s_U + \psi_{Uc}. \tag{9c}$$

Lemma 3 *The first-best can be decentralized with Pigovian taxes $t_i = \alpha_i\mu$ and $s_i = 0$*

This textbook results helps clarify two important points: if all emissions are taxed when emitted, then life cycle considerations are not required to design the optimal policy. Furthermore, there is no need to coordinate policies, each local regulator sets the same tax level. However, both of these points only hold when taxes are optimally set at the Pigovian level, a case rarely met in the real world, so it is worth investigating consequences of departure from this situation.

Before analyzing optimal couple of subsidies, let us look at the impact of each instruments on total emissions. Indeed, one would expect that taxes on dirty goods and subsidies on clean good both reduce total pollution.

Proposition 1 *At the market equilibrium:*

- *Total emissions always decrease with upstream subsidies or downstream taxes.*
- *Total emissions increase with respect to the subsidy on the clean downstream quantity if and only if $q_{Dc} > 0$ and*

$$\frac{\alpha_D}{1+\Gamma_{Dd}} < \frac{\theta\alpha_U}{\Gamma_{Ud}\left(1+\frac{1}{\Gamma_{Ud}}+\frac{1}{\Gamma_{Uc}}\right)} \tag{10}$$

- *Total emissions increase with respect to the tax on the dirty upstream quantity if and only if $q_{Ud} > 0$ and*

$$\alpha_U < \frac{\alpha_D}{1+\Gamma_{Dd}}\left[\theta + \frac{1}{\theta\Lambda}\left(1+\frac{1}{\Gamma_{Uc}}\right)\left(\Gamma_{Dc}+\frac{\Gamma_{Dd}}{1+\Gamma_{Dd}}\right)\right]^{-1} \tag{11}$$

Proof in Appendix B.1. To understand the mechanisms behind the Proposition 1, it is illuminating to consider the impact of a change of a quantity on the three others, rather than to look at the impact of the associated instrument (it is equivalent to use a price instrument or a quota to set a given quantity). Parameters Γ_{ij} and Λ, defined in equations (4), intervene in those changes. Any change of instrument modifies the equilibrium in both markets and thus the quantity of emissions in both sectors, the adjustment of the upstream market plays a crucial role.

The second point of the Proposition on the subsidy s_D on the clean downstream can be established by noting that, from equations (9a) and (9c), the three other quantities depend directly on the quantity q_{Dc} and only indirectly on the subsidy s_D. The change dq_{Dc} of the clean downstream quantity, generated by a change of s_D, is associated with the following adjustments of the other quantities:[9] In the Downstream sector, if $q_{Dd} > 0$,[10]

$$P'_D.[dq_{Dc}+dq_{Dd}] = C''_{Dd}.dq_{Dd} \text{ so } dq_{Dd} = -\frac{dq_{Dc}}{1+\Gamma_{Dd}}.$$

[9] In full rigor the equilibrium quantities should be defined as functions of the four instruments $q^E_{ij}(t_U, t_D, s_U, s_D)$, and the marginal variations dq_{ij} considered in the main text would be more rigorously written as $\partial q^E_{ij}/\partial s_D$ (see Appendix B.1). The analysis of the influence of t_U follows the same logic with $\partial q^E_{ij}/\partial t_U$.

[10] Actually, the formula for dq_{Dd} also holds for $q_{Dd} = 0$ by definition of Γ_{Dd} (cf equation 4a).

In the upstream sector

$$P'_U.[dq_{Ud} + dq_{Uc} - \theta dq_{Dc}] = C''_{Ud} dq_{Ud} = C''_{Uc} dq_{Uc}$$

and, dividing by $-P'_U$, the two upstream quantities are adjusted as follow

$$dq_{Uj} = \frac{\theta dq_{Dc}}{\Gamma_{Uj}\left(1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}\right)} \text{ for } j = d, c$$

The inequality (10) is then a comparison between the emissions avoided in the downstream sector (left-hand side) with the emissions generated in the upstream sector (right-hand side) by an additional unit of the clean downstream. Concerning corners situations, if $q_{Dd} = 0$ the inequality is satisfied since the left-hand side is null ($\Gamma_{Dd} = +\infty$), and if $q_{Ud} = 0$ it is not, since the right-hand side is null ($\Gamma_{Ud} = +\infty$).

And finally the third point of Proposition 1 could be interpreted as a comparison between the emissions generated in the upstream sector (left-hand side) and avoided in the downstream sector (right-hand side) by an additional upstream dirty unit. Indeed, in the upstream sector an additional dirty unit generates α_U tCO_2. In the downstream sector, the adjustment of quantities is mediated through the upstream price p_U (from equations (9b) and (9a)):

$$dq_{Dd} = \frac{-1}{1 + \Gamma_{Dd}} dq_{Dc} = \frac{1}{\Gamma_{Dd}\Gamma_{Dc} + \Gamma_{Dc} + \Gamma_{Dd}} \frac{\theta dp_U}{-P'_D}$$

The second equality exhibits the slope of the demand for the upstream good emanating from the downstream sector. An additional dirty upstream unit reduces the upstream price which increases the clean downstream quantity and reduces downstream emissions. And, in the upstream sector (taking the derivative of equation (9c)), the following relationship between adjustments of quantities must hold:

$$P'_U.[dq_{Ud} + dq_{Uc} - \theta dq_{Dc}] = dp_U = C''_{Uc}.dq_{Uc}$$

And, after some manipulations (cf Appendix B.1) inequality (11) follows. The right-hand side corresponds to the reduction of emissions in the downstream sector generated by an additional dirty upstream unit, the first factor is the rate of the substitution between the dirty and clean downstream quantities, the second factor encompasses the adjustment of the upstream price.

An increase of the tax on the dirty upstream variety is more likely to increase emissions if the downstream emission intensity α_D is large, the dirty downstream quantity is highly responsive to the clean downstream quantity (Γ_{Dd} small), and the upstream demand and upstream clean supply are price inelastic ($-P'_U$ and Γ_{Uc} large).

From these comparative statics one can deduce the impact of the SCC on the optimal allocation as summarized in the following corollary.

Corollary 1 *A marginal increase of the SCC induces the following changes of the optimal allocation:*

- *Total and downstream emissions decrease, upstream emissions increase if and only if* $q^*_{Ud} > 0$ *and condition (11) holds.*
- *Quantities consumed by households decrease in both sectors.*
- *The clean upstream quantity increases, and, the clean downstream quantity decreases if and only if* $q^*_{Dc} > 0$ *and condition (10) holds.*

The proof is in Appendix B.1. Most consequences of an increase of the SCC are intuitive except for dirty upstream and clean downstream quantities. Dirty upstream production increase if condition (10) is satisfied, and downstream clean production decreases if condition (11) is satisfied. When the SCC increases both the demand for and the cost of the clean downstream technology increases and if condition (10) is satisfied the latter dominates and the clean downstream quantity decreases. In the upstream sector, the cost of the dirty technology increases with the SCC but the demand for the upstream good, emanating from the downstream sector, increases and, if condition (11) holds, can compensates for the cost increase and requires an expansion of the dirty technology. The two conditions (10) and (11) are mutually exclusive,[11] the increase of upstream dirty production occurs only if the downstream clean technology expands.

If dirty technologies have linear costs, $\Gamma_{id} = 0$ for $i = U, D$, if dirty technologies are used they should set the price in both sectors ($p_i = c_{id} + \alpha_i \mu$), and the two conditions (10) and (11) are simplified. Notably condition (10), under which the clean downstream decreases with respect to the SCC, is $\alpha_D < \theta \alpha_U$, which contradicts Assumption 2.

The influence of the sector coupling intensity θ is interesting. While condition 10 is linear in θ, it is more likely to hold the larger θ is. Condition 11 holds for intermediary values of θ is small the deployment of the clean downstream does not require a sufficient amount of the upstream good to trigger an expansion of the dirty upstream quantity. If θ is large, the cost of the clean downstream strongly increases with respect to the SCC which limits the expansion of the clean downstream and thus the need for more upstream production.

4.2 Optimal downstream policies

4.2.1 Optimal subsidy

Let us start with a discussion of the optimal downstream subsidy for a given tax on the dirty downstream technology and regulation (both tax and subsidy) in the upstream sector. For instance, in the case of electric mobility, the question is whether emissions associated with electricity production should influence the optimal subsidy on electric vehicles. Even though the subsidy is initially justified by the unpriced negative externality from the dirty downstream technology, it should also be adjusted to the suboptimal upstream regulation.

For an instrument τ (a tax or a subsidy), the maximization of the welfare function given by equation (1) gives the first-order condition :

[11] Multiplying both sides of 10 by θ gives $\alpha_D/(1 + \Gamma_{Dd}) > \alpha_U$, while condition 11 implies $\alpha_D/(1 + \Gamma_{Dd}) < \alpha_U$.

$$0 = \sum_{i,j} \frac{\partial W}{\partial q_{i,j}} \frac{\partial q_{i,j}}{\partial \tau} \tag{12}$$

Injecting equations (9a), (9b), (9c), satisfied at the market equilibrium gives:

$$s_U \frac{\partial q_{Uc}}{\partial \tau} + (\alpha_U \mu - t_U) \frac{\partial q_{Ud}}{\partial \tau} + s_D \frac{\partial q_{Dc}}{\partial \tau} + (\alpha_D \mu - t_D) \frac{\partial q_{Dd}}{\partial \tau} = 0 \tag{13}$$

for all instruments, each derivative $\frac{\partial q_{ij}}{\partial \tau}$ only depends on θ, Γ_{ij} and Λ (defined in equations 4), which enables to obtain an explicit formula for the optimal downstream subsidy, as given by the following proposition.

Proposition 2 *For given downstream tax t_D and upstream tax and subsidy t_U and s_U, the optimal downstream subsidy, is:*

$$s_D = \frac{1}{1 + \Gamma_{Dd}} (\alpha_D \mu - t_D) - \frac{\theta}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \left[(\alpha_U \mu - t_U) \frac{1}{\Gamma_{Ud}} + s_U \frac{1}{\Gamma_{Uc}} \right] \tag{14}$$

The proof is in Appendix B.3. The optimal downstream subsidy is justified by unpriced externalities, indeed, if the externality is taxed at the Pigovian level, so $t_i = \alpha_i \mu$ and $s_U = 0$, the optimal subsidy is null. In the downstream sector, an increase of clean production reduces dirty production by an amount determined by the slopes of consumer demand and of dirty marginal cost. If either the demand is inelastic or dirty cost are linear the rate of substitution is equal to minus one.[12]

Concerning the influence of the upstream sector regulation: First, if the externality is perfectly priced in the upstream sector, ($t_U = \alpha_U \mu$, $s_U = 0$) the emission intensity of the upstream sector does not intervene in the formula. It is so because the environmental cost is already encompassed in the upstream price. Second, the optimal downstream subsidy does not depend on the average mix in the upstream sector but on the emission intensity of the marginal unit which is a weighted sum of dirty and clean production, the weights depending on the slope of the respective marginal costs. With a linear dirty upstream cost ($\Gamma_{Ud} = 0$) that marginal unit is dirty as long as there is some dirty production, and, in such a case, the optimal downstream subsidy is:

$$s_D = \frac{1}{1 + \Gamma_{Dd}} (\alpha_D \mu - t_D) - \theta (\alpha_U \mu - t_U). \tag{15}$$

A similar result to Proposition 2 could be established for the upstream subsidy. The optimal upstream subsidy incorporates terms related to the suboptimal regulation of the downstream sector.

[12] The formula could be generalized to take into consideration an imperfect substitution between dirty and clean downstream goods on the consumer side as empirically investigated by Xing et al. (2021).

4.2.2 Consequential LCA

While conventional LCA focuses on the physical flows that composes the life cycle of a product (from material extraction to end of life), consequential LCA aims at including the marginal adjustment of equilibrium quantities (Rajagopal and Zilberman, 2013; Rajagopal, 2014). Using a LCA to assess the impact on emissions of adding a unit of a good is valid if other quantities are maintained fixed, a consequential LCA remedies to that flaw. Proposition 1 can be interpreted as a stylized consequential LCA of each of the four goods present in our model. And Corollary 1 states that a technology with positive consequential emissions should be less used as the external cost of emissions increase. Therefore, LCA are linked to static considerations, that is, the computation of MAC and determination of the optimal allocation, whereas consequential LCA are linked to the evolution of quantities along the transition.

The result that a good (whether an input or a consumption good) with positive consequential emissions should be less produced as the external cost of emissions increases, is general and not specific to our model. First, the optimal allocation corresponds to the market equilibrium with Pigovian taxes. Second, the consequential emissions of a good can be interpreted as an assessment of the substitutability of a good with total emissions, a negative (resp. positive) consequential emissions means that the two are substitutes (resp. complements). And, as the external cost of emissions increase, goods that are substitutes to emissions should be produced more and complements produced less (cf the proof of Corollary 1 in Appendix B.1).

Concerning the optimal subsidies, the formula (14) in Lemma **??** for the optimal downstream subsidy can be linked to life cycle considerations. If all three other instruments are null the optimal subsidy is

$$s_D = \mu \left[\alpha_D \frac{1}{1+\Gamma_{Dd}} - \theta \alpha_U \frac{1}{\Gamma_{Ud}} \frac{1}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \right]$$

which correspond to the SCC times the consequential emissions from an additional unit of the clean downstream. The first term corresponds to the emissions avoided in the downstream sector and the second one the emissions generated in the upstream sector (cf discussion below Proposition 1). With non-null tax and subsidies, the formula should be corrected to only account for external costs.

4.2.3 Alternative upstream instruments

In practice, in many countries, multiple regulations are in place in candidate upstream and downstream sectors, notably in the electricity, transportation, industry and building sectors. For instance, in the EU national electricity sectors, and some industrial sectors, are covered by the EU Emission Trading Scheme, and low carbon technologies (renewable and nuclear where it is still developed) are subsidized with targeted shares of renewable in total electricity production.[13] Furthermore, in both the US and the EU, transportation sectors are subject

[13]Concerning the EU-ETS, it is a cap and trade system with several additional features, most notably the market stability reserve that makes the cap flexible, total emissions are therefore more or less fixed.

to several regulations: an EU standard on fossil cars emission intensity and subsidies on electric cars.

Here, we do not aim at an exhaustive analysis of second-best instruments coordination, and only explore the optimal downstream subsidy for a given general upstream regulation. Indeed, the adjustments of quantities in the upstream sector influence the optimal downstream subsidy, and these adjustments could be described with supply curves. Proposition 3 enunciates the principle of optimal second-best downstream subsidy.

Proposition 3 *The optimal downstream subsidy is the difference between the marginal external benefit from reduced downstream emissions and the sum of the adjustments of upstream quantities to face the additional demand weighted by their respective implicit subsidies.*

These implicit subsidies relate to the mispricing of pollution and subsidy of the upstream clean technology, and depends on the upstream regulation.

Let us formalize, and prove Proposition 3. Consider that the upstream regulation is fixed and upstream quantities adjust according to the supply curves $q^r_{Uj}(p_U)$ for $j \in \{d, c\}$. The upstream market equilibrium is described by the market clearing equation:

$$q^r_{Ud}(p_U) + q^r_{Uc}(p_U) = P_U^{-1}(p_U) + \theta q_{Dc}.$$

Any increase of the downstream clean quantity is associated with an adjustment on the upstream market described by the two equations :

$$dq_{Uj} = q^{r\prime}_{Uj} dp_U = q^{r\prime}_{Uj} \frac{\theta dq_{Dc}}{q^{r\prime}_{Uc} + q^{r\prime}_{Ud} - 1/P'_U} \text{ for } j = d, c.$$

By analogy with Γ_{Uj}, we note $\Gamma^r_{Uj} = -1/(P'_U q^{r\prime}_{Uj})$, the ratio of the slopes of the supply of technology $j = d, c$ and upstream demand, gives a generalization of the formula (14) satisfied by the optimal subsidy:

$$s_D = \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \frac{\theta}{1 + \frac{1}{\Gamma^r_{Ud}} + \frac{1}{\Gamma^r_{Uc}}} \left[(\alpha_U \mu - (p_U - C'_{Ud})) \frac{1}{\Gamma^r_{Ud}} + (C'_{Uc} - p_U) \frac{1}{\Gamma^r_{Uc}} \right]. \tag{16}$$

The downstream subsidy needs be reduced by the indirect subsidy encompassed in the upstream price. The second term above is the sum of the implicit subsidies, in parenthesis, times the adjustment of the associated quantity.

The parameters $\Gamma^{r\prime}_{Uj}$ depend on the regulation in the upstream sector. The following Corollary illustrates this result with an upstream mandate.

Corollary 2 *If there is a mandate of a (binding) share r_U of clean production in the upstream sector ($q_{UC} = r_U(q_{Ud} + q_{Uc})$), the optimal downstream subsidy is:*

$$s_D = \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \theta \frac{(1 - r_U)\alpha_U \mu}{1 + r_U^2 \Gamma_{Uc} + (1 - r_U)^2 \Gamma_{Ud}} \tag{17}$$

The proof is in Appendix B.4. With an upstream mandate, the share of the dirty upstream production is fixed at $(1-r_U)$, and $\alpha_U(1-r_U)$ is then the average emission intensity of upstream production. The optimal downstream subsidy should encompasses these upstream emissions weighted by the reduction of upstream demand. Indeed, if upstream demand is inelastic, or upstream costs are linear, then $\Gamma_{Uj} = 0$ for $i = d, c$ and formula (17) simplifies to

$$s_D = \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \theta(1 - r_U)\alpha_U \mu.$$

In that specific situation (a mandate and inelastic demand), the optimal downstream subsidy should be corrected by the average upstream emissions.

4.3 Policy coordination with imperfect carbon pricing

This section investigate the second-best policies in the two sectors when carbon pricing is unavailable. We start by computing the second-best policies in the two sectors when taxes are bounded in both sectors. With the specification we are able to compare quantities and welfare between first-best and second-best policies. Finally, we briefly discuss the impact of imperfect carbon pricing in only one sector, downstream or upstream.

4.3.1 Second-best subsidies

The following Proposition provides the formula of optimal subsidies that are jointly optimized.

Proposition 4 *For given taxes t_D and t_U, the optimal two second-best subsidies s_D^{SB} and s_U^{SB} satisfy the following equations, if $q_{id} > 0$ in both sectors $i = U, D$,*

$$s_D^{SB} = \frac{1}{1 + \Gamma_{Dd}}\big(\alpha_D \mu - t_D\big) - \theta \frac{1}{1 + \Gamma_{Ud}}\big(\alpha_U \mu - t_U\big) \tag{18a}$$

$$s_U^{SB} = \frac{1}{1 + \Gamma_{Ud}}\big(\alpha_U \mu - t_U\big) \tag{18b}$$

While the optimal downstream subsidy still encompasses elements from the upstream sector, it is not so for the upstream subsidy.[14] The optimal upstream subsidy is only determined by substitution between clean and dirty production in the upstream sector but not in the downstream sector. The ratio Γ_{Ud} only encompasses such local sector substitution and not the adjustment of demand emanating from the upstream sector. It is so because the downstream subsidy optimally adjusts and absorbs change of the upstream price. There is an asymmetry between the two sectors because a subsidy on the clean downstream good rises the demand for the upstream good whether clean or dirty, whereas a subsidy on the clean upstream good has only an impact on the supply of the clean downstream technology,

[14]It is not exactly true in full rigor since the upstream sector characteristics indirectly influence ϵ_i in the general model, but not in a quadratic specification.

and not the dirty one. That asymmetry explains that the regulation of the upstream sector does not need to consider downstream regulation (and it could easily be extended to several downstream sectors), whereas the downstream regulation should take into account upstream considerations.

The above optimal couple of subsidies is obtain for an interior situation in which all technologies are used. In that case, the substitution between clean and dirty production plays a crucial role, the motivation for the subsidies being precisely to reduce dirty production. However, for sufficiently large SCC the two sectors are eventually decarbonized, in that case the subsidies are used to ensure that dirty production is not profitable. Of particular interest is the case in which the upstream sector is decarbonized but the downstream is not. This case will arise in our numerical illustration. The following lemma characterizes the optimal couple of subsidies in that case.

Lemma 4 *The optimal couple of subsidies satisfies, if $q_{Ud} = 0$ and $q_{Dd} > 0$ at the second-best:*

$$s_D^{SB} = \frac{1}{1+\Gamma_{Dd}}\big(\alpha_D \mu - t_D\big) - \theta s_U^{SB} \tag{19}$$

$$\begin{aligned} s_U^{SB} &= C'_{Uc}(q_{Uc}) - [C'_{Ud}(0) + t_U] \\ &= C'_{Uc}(q_{Uc}) - P_U(q_{Uc} - \theta q_{Dc}) \end{aligned} \tag{20}$$

The Proof if in Appendix B.5. The downstream subsidy formula is familiar, it is the difference between avoided emissions in the downstream sector, and the upstream subsidy. Even though the upstream sector is fully decarbonized the downstream subsidy needs be reduced by the implicit subsidy in the upstream price. The upstream subsidy does not directly depends on the SCC, it is set to keep dirty upstream production unprofitable. The upstream price is equal to the marginal cost of the dirty technology at zero : $p_U = C'_{Ud}(0) + t_U$. Therefore, as the SCC keeps increasing the downstream clean technology expands, and to face that additional demand the upstream subsidy also increases.

With the quadratic specification 1, one can get explicit expressions of equilibrium and optimal (first-best and second-best) quantities, and also of the welfare loss of the second best policy compared to the first best.

Corollary 3 *Given specification 1, and two taxes t_U and t_D, at the second-best policy, if all quantities are positive:*

- *The clean quantities in the upstream and downstream sectors are equal to their first-best values; the dirty quantities are larger than their first-best values.*
- *The welfare loss between the first-best and the second-best policy does not depend on sectoral linkage, it is:*

$$W^{FB} - W^{SB} = \frac{1}{2}\frac{1}{1+\Gamma_{Ud}}\frac{(\alpha_U \mu - t_U)^2}{b_U} + \frac{1}{2}\frac{1}{1+\Gamma_{Dd}}\frac{(\alpha_D \mu - t_D)^2}{b_D}. \tag{21}$$

- *The welfare of the upstream sector is always higher in the first-best than in the second-best. In contrast, the welfare of the downstream sector may be higher in the second-best than in the first-best:*

$$W_U^{FB} - W_U^{SB} = \frac{1}{2}\frac{1}{1+\Gamma_{Ud}}\frac{(\alpha_U\mu - t_U)^2}{b_U} - \theta s_U q_{Dc} \tag{22a}$$

$$W_D^{FB} - W_D^{SB} = \frac{1}{2}\frac{1}{1+\Gamma_{Dd}}\frac{(\alpha_D\mu - t_D)^2}{b_D} + \theta s_U q_{Dc} \tag{22b}$$

The proof is provided in Appendix B.6. It is not *a priori* straightforward to compare second-best and first-best clean quantities, and it is remarkable that they coincide with the quadratic specification. There are two opposite effects: a satiation effect and a substitution effect. With above optimal dirty quantities, clean production is less necessary to satisfy consumers (satiation effect) but are used to substitute for the dirty technology (substitution effect). In the quadratic specification these two effects compensate exactly (cf Appendix B.6).

Welfare loss given by equation (21) is quadratic in the absolute mispricing of carbon emissions in each sector $\alpha_i\mu - t_i$. The slopes of the demand function and the dirty production function intervene in the welfare loss. The larger these slopes the lower the loss. These slopes can be interpreted as a measure of the elasticity of demand and dirty supply. At the extreme, with inelastic demands or inelastic dirty supply functions the second best subsidies can mimic the first best. The welfare loss is independent from θ, it is a peculiarity of the quadratic specification, linked to the first point of the Lemma.

The last point of the corollary compares welfare at the sectoral level between first-best and second-best policies. Differences (22) in sectoral welfare are the sum of a term from carbon mispricing and a transfer $\pm\theta s_U^{SB} q_{Dc}$ from the upstream to the downstream sector related to the clean upstream subsidy. The latter may be interpreted in two ways. First, it translates the implicit subsidy from the upstream sector to the clean downstream technology. Second, it is equal to the cost of mispriced emissions indirectly emitted by the clean downstream technology. Surprisingly, the downstream sector is better off with the second-best policy than with the first-best one if the clean quantity is sufficiently large:

$$q_{Dc} > \frac{1}{2\theta b_D}\frac{1+\Gamma_{Ud}}{1+\Gamma_{Dd}}\frac{(\alpha_D\mu - t_D)^2}{\alpha_U\mu - t_U}.$$

The numerical illustration will exhibit such situation.

4.3.2 Imperfect carbon pricing in a single sector

To conclude the analysis of second-best policies, we consider cases in which the tax on the dirty technology is bounded in a single sector. The following Corollary describes the optimal policy in those cases. The absence of a sufficient tax in one sector justifies to subsidize the clean alternative, in the other sector a Pigovian tax can be implemented.

Corollary 4 *With imperfect carbon pricing in only one sector:*

- *If the downstream tax t_D is bounded but not the upstream tax t_U then the optimal policy consists in:*

$$t_U = \alpha_U \mu \text{ and } s_U = 0 \tag{23a}$$
$$s_D = \frac{1}{1+\Gamma_{Dd}}\big(\alpha_D \mu - t_D\big) \tag{23b}$$

- *If the upstream tax t_U is bounded but not the downstream tax t_D, and the clean downstream technology can be taxed, then the optimal policy consists in:*

$$s_U = \frac{1}{1+\Gamma_{Ud}}\big(\alpha_U \mu - t_U\big) \tag{24a}$$
$$t_D = \alpha_D \mu \text{ and } s_D = -\theta s_U \tag{24b}$$

- *If the upstream tax is bounded but not the downstream tax t_D and the clean downstream technology cannot be taxed ($s_D \geq 0$), then the optimal policy consists in:*

$$s_U = \frac{\alpha_U \mu - t_U}{1 + \Gamma_{Ud} + \theta^2 \Lambda \frac{\Gamma_{Ud}}{1+\Gamma_{Dc}}} \tag{25a}$$
$$t_D = \alpha_D \mu - \theta s_U \frac{1}{1+\Gamma_{Dc}} \text{ and } s_D = 0 \tag{25b}$$

If carbon pricing is constrained only in the downstream sector, the optimal upstream tax is Pigovian and there is no need to subsidize the upstream clean technology. A direct consequences is that the optimal downstream subsidy, from lemma **??**, does not encompass any upstream consideration since upstream externality are perfectly internalized. Conversely, if carbon pricing is constrained in the upstream sector it is then optimal to subsidize the upstream clean technology and the optimal downstream regulation consists in a Pigovian tax together with a tax on the downstream clean technology to correct for the indirect subsidy due to the upstream subsidy. If the clean downstream cannot be taxed then the upstream subsidy is lower, the denominator in equation (25a) encompasses the adjustment of the clean downstream quantity and the downstream tax is reduced to compensate for the upstream subsidy.

5 Numerical illustration

5.1 Calibration and solving

This section aims to illustrate the two previous parts on social optimum and on the coordination of sectoral policies with a calibrated numerical example, inspired by the electrification of passenger cars in France for sector sizes and cost but with a hypothetical electricity mix comprised of gas-fired plants and renewables.[15] It will also provide an example of a full transition of both sectors. Some relevant data are shown on table 1. We detail the calibration

[15] The aim is to illustrate the theoretical results obtained through a stylized simulation, we abstract from many relevant issues, most notably: the regulated price of electricity, the partial interconnection of European power systems, the uncertain future of nuclear power, the variability of electricity demand and car charging, and the actual policies in both sectors among which the EU-Emission Trading Scheme.

	Upstream (U)	Downstream (D)
a_i	395 €/MWh	1.215 €/km
b_i	0.46 10^{-6}€/MWh^2	1.63 10^{-9}€/km^2
c_{id}	177 €/MWh	0.5 €/km
α_i	0.35 tCO_2/MWh	0.12 $kgCO_2$/km
θ		0.2 kWh/km
c_{ic}	180 €/MWh	0.467 €/km
Γ_{ic}	0.75	0.035

Table 1: Parameter values

of the demand and costs functions in Appendix C.[16] We consider gas-fired plants ($\alpha_U = 0.35$ tCO_2/MWh) and gasoline engines ($\alpha_D = 0.12$ tCO_2/km) as dirty technologies, and renewable and electric vehicles as clean technologies. We assume that both technologies do not directly emit CO_2. We assume that the linkage coefficient is given by the typical energy efficiency of the engine of electric vehicle ($\theta = 0.2kWh/km$).

Our numerical illustration is divided into two distinct parts. First, we simulate four policies in a static context with a SCC of 150€/tCO_2:

- BAU: Market equilibrium with neither taxes nor subsidies;
- FB: Market equilibrium with Pigovian taxes;
- SB: Second-best subsidies with imperfect carbon taxation with taxes equal to 20% of the Pigovian level;
- SBcap: Second-best subsidies with imperfect carbon taxation and total emissions equal to emissions in FB, and same taxes as in SB.

Second, the model is simulated for a range of social costs of carbon going from 0 to 600€/tCO_2 where we focus on the first three policies (BAU, FB and SB).

5.2 Quantities, Emission and surpluses

Effect of instruments on emissions With our parameter set, total emissions are decreasing with respect to the downstream subsidy and the upstream tax.

For total emissions to increase with respect to the downstream subsidy one needs $\theta\alpha_U > \alpha_D$ (from equation (10) with $\Gamma_{id} = 0$) which is largely the case for coal-power electricity. However, if $\theta\alpha_U > \alpha_D$ the transition would be sequential with first the decarbonization of electricity and then of transportation, and, at the second-best optimum, electric cars would not be subsidized until electricity is fully decarbonized.

For total emissions to decrease with respect to t_U condition (11) needs be satisfied and with Γ_{id} being null it amounts to:

$$\alpha_U \left[\theta + \frac{b_U}{\theta b_D}(1 + \frac{1}{\Gamma_{Uc}})\Gamma_{Dc}\right] < \alpha_D.$$

[16] The model is solved using complementarity methods with disjunctive constraints as described in Gabriel et al. (2012). The python code is available upon request.

Our parameter values are such that $\alpha_U < \alpha_D/\theta$ ($0.35 < 0.6$), so that with a sufficiently large, or elastic, downstream sector (small b_U) together with flat clean downstream cost and steep clean upstream ones, the condition would be satisfied.[17] Adding other downstream sectors (heavy transport, heating) together with a steeper upstream clean supply would increase the probability of this situation. However, the cost of the clean downstream technologies would need be sufficiently flat which is unlikely.

Policy	BAU		FB		SB		SBcap	
Sector	U	D	U	D	U	D	U	D
Q_i (TWh, 10^9 km)	473	436	361	425	450.5	433.8	450.5	433.8
q_{ic}	0	0	139	87	139	87	222	159
q_{id}	473	436	239	338	329	347	260.54	274.37
P_i (€/MWh, 10^{-3} €/km)	177	500	229.5	518	187.5	503	187.5	504
t_i	0	0	52.5	18	10.5	3.6	10.5	3.6
s_i	0	0	0	0	42	6	71	10.1

Table 2: Quantities and prices

Quantities and prices Table 2 gives the quantities and price in each sector for the four scenarios considered. In BAU, there is no clean production. In FB, Pigovian taxes reduces total production and stimulates clean production. The impact on consumer prices is much higher in the upstream sector than in the downstream sector (respectively 29% and 4%) because of the difference in emission rates over prices α_i/c_{id}. In SB, the taxes are only 20% of their FB levels, so the quantities consumed are close to their BAU levels. As shown in Lemma 3, clean quantities are the same in FB and SB. As shown by Proposition 4, in SB the upstream subsidy is equal to the difference between the Pigovian tax and the actual tax, while in the downstream sector, the second-best subsidy is lower because of mispricing in the upstream sector. In SBcap, demand and prices are the same as in SB, but subsidies are twice larger to reach the same cap as in FB.

Emission and abatement Table 3 presents the total and sectoral surpluses and emissions in the three scenario. We are interested in the allocation of efforts among the two sectors, both in terms of abatement and consumers surplus. It is noteworthy that the upstream sector represents almost 70% of total emissions but only 15% of total welfare.

[17]For instance, if the slope of the clean upstream is multiplied by 2 and the slope of the clean downstream divided by 2 then the condition would be satisfied with a downstream sector at 2.2 times larger.

	BAU	FB	%FB/BAU	SB	%SB/BAU	SBcap	%SBcap/BAU
W	175.4	182.0	3.8	180.1	2.7	178.7	1.9
CS	208.0	178.4	-14.2	201.6	-3.1	201.6	-3.1
$W + \mu E$	208.0	200.7	-3.5	203.6	-2.1	197.4	-5.1
E	217.9	124.1	-43.0	156.6	-28.1	124.1	-43.0
W_U	27.5	33.9	23.1	31.2	13.6	29.4	7.1
CS_U	52.3	30.4	-41.8	47.5	-9.3	47.5	-9.3
$W_U + \mu E_U$	52.3	46.4	-11.3	48.5	-7.3	43.1	-17.6
E_U	165.5	83.6	-49.5	115.0	-30.5	91.2	-44.9
W_D	147.9	148.2	0.2	148.9	0.7	149.3	1.0
CS_D	155.7	148.0	-5.0	154.1	-1.0	154.1	-1.0
$W_D + \mu E_D$	155.7	154.3	-0.9	155.1	-0.4	154.2	-0.9
E_D	52.3	40.5	-22.5	41.6	-20.5	32.9	-37.1
E_{LCA} [a]	0.0	3.9	-	4.3	-	6.0	-
E_{CLCA} [b]	0.0	6.1	-	6.1	-	11.1	-

[a] Attributional life cycle emission from the clean downstream technology: $E_{LCA} = \theta q_{Dc} \times \alpha_U q_{Ud}/(q_{Uc}+q_{Ud})$
[b] Consequential life cycle emission from the clean downstream technology: $E_{CLCA} = \theta \alpha_U q_{Dc}$

Table 3: Sectoral and total surpluses (in M€) and emissions (in $MtCO_2$), with their variation (in %) relatively to the business-as-usual case (BAU)

Sector	Abatement source	FB	SB	SBcap
Upstream	Consumption	43	13	8.5
	Clean	52	79.5	82.5
	Total	94	92.5	91
Downstream	Consumption	1.5	0.5	0.5
	Clean	11	17	20.5
	Consequential	-6.5	- 10	-12
	Total	6	7.5	9
Total		100	100	100

Table 4: Allocation of Abatement (in %). Total abatement in each scenario is decomposed as the sum of demand reduction and clean technology production: $E^0 - E = \alpha_U(Q_U^0 - Q_U) + \alpha_U q_{Uc} + \alpha_D(Q_D^0 - Q_D) + \alpha_D q_{Dc} - \alpha_U \theta q_{Dc}$, the last term being consequential life-cycle emissions.

Table 4 decomposed the effort among sectors and the two channels: consumption reduction and clean technology deployment. In FB, 43% of emissions are abated. Most of abatement is done in the electricity sector, that cuts half of its emissions, an effort that is nearly evenly allocated between a reduction of consumption and a deployment of clean electricity. In SB, only 25% of emissions are cut, since demand reductions are less solicited, the effort is slightly reallocated to mobility, the demand of which being the less elastic. In SBcap clean technologies are further mobilized and the share of effort of the downstream sector is higher. We define life-cycle emissions from electric cars with two metrics, the first corresponds to standard attributional LCA and is computed with the average emission intensity from the electricity sector $q_{Ud}/(q_{Uc}+q_{Ud})$, and the second corresponds to a CLCA approach and uses the marginal emission intensity $\theta\alpha_U$. LCA emissions account for a limited share of upstream emissions, around 5%, but close to 20% of downstream emissions in SBcap. As the upstream sector is more polluting in average in SB than in FB, LCA emissions increase in SB compared to FB. Indeed, CLCA emissions are larger than LCA ones, and the more so the cleanest the upstream technology mix.

Surpluses Overall, welfare gain from environmental policies are small in magnitude with 4% in FB and 2% in SB. However, welfare gains in the upstream sector are large with 23% in FB and 13% in SB, compared to respectively 0.2% and 0.7% in the downstream sector. Surprisingly, the welfare in the downstream sector is higher in SB compared to FB, and even more so in SBcap. Indeed, without sector interaction, the reverse would necessarily hold. With sector interaction, the downstream clean technology benefits from the lack of upstream taxation. There is an implicit subsidy encompassed into the price of the upstream good. The amount $\theta s_U q_{Dc}$, which is equal to $\theta(\alpha_U\mu - t_u)q_{Dc}$, is a transfer from the upstream to the downstream sector in SB, and SBcap, compared to FB. Given the relatively small contribution of consumption reduction in the downstream sector, this transfer explains nearly all the 0.5% gain between SB and FB. This effect is larger the stronger the linkage intensity.

5.3 Transition

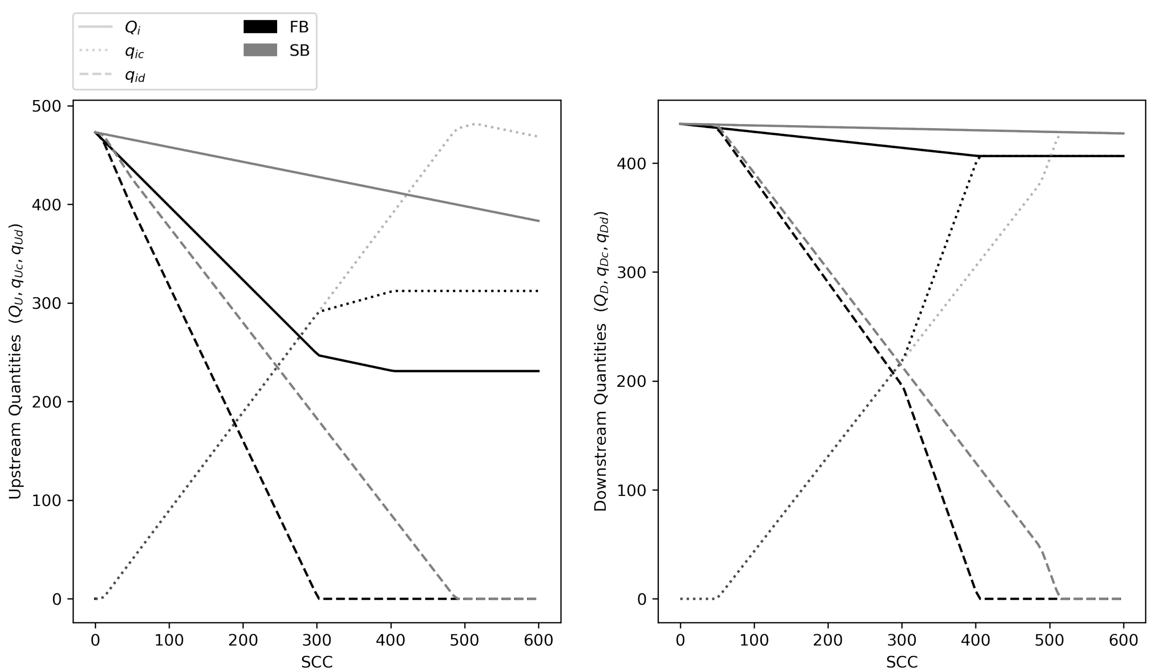

Figure 2: Evolution of upstream (right) and downstream (left) quantities with social cost of carbon for FB and SB policies. Note that FB and SB clean quantities are equal until the decarbonization of the sector (cf Corollary 3)

To investigate the difference of policies in a transition towards a fully decarbonized system. Figure 2 shows the evolution of quantities (demand, clean and dirty production) with respect to the SCC.

The upstream sector is the first to be fully decarbonized in both FB or SB policies. There is an acceleration of the deployment of the clean downstream technology once the upstream sector is fully decarbonized. In FB, upstream prices aligns with the clean technology costs instead of the dirty costs plus the growing SCC. Slower growth of upstream prices eases the deployment of the downstream clean technology. In SB, the same acceleration is explained by the form of the upstream subsidy described in Lemma 4. Indeed, the subsidy equates the cost differential between the clean and dirty upstream technology. As the upstream carbon tax (slightly) increases, the upstream subsidy decreases, and the downstream subsidy increases.

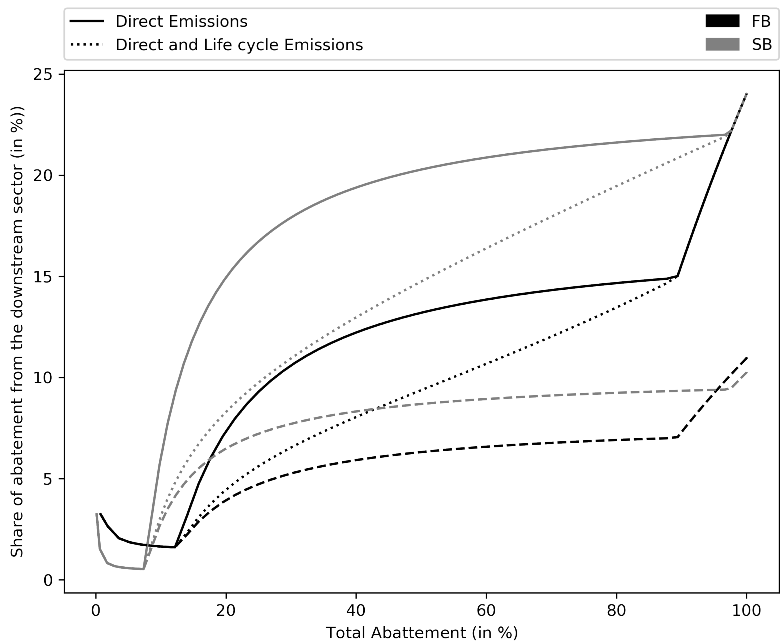

Figure 3: Share of abatement coming from the downstream sector versus the overall abatement. Full lines do not include life cycle emissions, dotted lines do with $E_{LCA} = \theta q_{Dc} \times \alpha_U q_{Ud}/(q_{Uc} + q_{Ud})$ attributed to the downstream sector.

Finally, we investigate how abatement shares evolves during the transition for FB and SB policies. Figure 3 highlights three aspects. First, the abatement from the downstream sector has a larger share in SB compared to FB almost all along the transition. Second, both shares evolves similarly, with a plateau until the decarbonization of the upstream sector. At this plateau, both sectors have a similar pace of decarbonization. Third, indirect emissions E_{LCA} is much higher in SB than in FB. Moreover, they can be of comparable size compared to the abated downstream emission. For instance, for a total abatement of 25%, excluding indirect emissions lowers the share of downstream abatement by a third.

6 Conclusion

We analyzed the coordination of sectoral decarbonization policies in an economy with interconnected sectors. Such issues are particularly important in the debate on the carbon footprint of electric vehicles, and for other electrification options (e.g. heating, cultured meat), and outside electricity for hydrogen and biogas deployments. We focused on the influence of carbon mispricing on the design of downstream subsidies and the coordination between downstream and upstream policies.

The analysis of second-best subsidy in the downstream sector stressed three main points: only unpriced externalities influence the optimal subsidy, the marginal upstream unit and not the average one matters, and, along a decarbonization transition the optimal downstream subsidy should evolve depending on the state of the upstream sector. Subsidies on clean technologies in both sectors should be coordinated. At the optimal second best policy, the upstream subsidy does not incorporate features of the downstream sector whereas the optimal downstream subsidy should be reduced to account for the second-best upstream policy.

This work could be improved in several ways. First, even though we used our model to describe a dynamic transition with an increasing SCC, it is fundamentally static, and should be extended into a dynamic framework taking into account inertia and technical change. Second, our policy framework assumes that sectoral regulations are designed by a single entity. However, this might not be the case in federal systems where the upstream sector is regulated at the national level and downstream sector at the state level. Nothing guarantees that the welfare functions of these regulators are aligned. Our model could be easily applied. Finally, our analysis of second best policies could be improved by introducing explicit constraints on carbon pricing in the spirit of the work of Galinato and Yoder (2010) to better understand how such constraints transfer efforts among interconnected sectors.

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A Quadratic specification

We provide here the expressions of quantities in the two situations considered in Assumption 1 with Specification 1.

With only dirty technologies :

$$Q_i^0 = \frac{1}{b_i(1+\Gamma_{id})}(a_i - c_{id}) \text{ and } p_i^0 = \frac{c_{id} + \Gamma_{id} a_i}{1 + \Gamma_{id}}.$$

With only clean technologies, the two quantities Q_U^1 and Q_D^1 if positive satisfy the couple of equations

$$\begin{aligned} a_U - b_U Q_U &= c_{Uc} + b_U \Gamma_{Uc}(Q_U + \theta Q_D) \\ a_D - b_D Q_D &= c_{Dc} + b_D \Gamma_{Dc} Q_D + \theta\big(c_{Uc} + b_U \Gamma_{Uc}(Q_U + \theta Q_D)\big); \end{aligned}$$

the unique solution of which is:

$$\begin{aligned} Q_U^1 &= \frac{1}{\Delta}\left[(b_D(1+\Gamma_{Dc}) + b_D\Gamma_{Dc} + \theta^2 b_U \Gamma_{Uc})(a_U - c_{Uc}) - \theta b_U \Gamma_{Uc}(a_D - c_{Dc} - \theta c_{Uc})\right] \\ Q_D^1 &= \frac{1}{\Delta}\left[b_U(1+\Gamma_{Uc})(a_D - c_{Dc} - \theta c_{Uc}) - \theta b_U \Gamma_{Uc}(a_U - c_{Uc})\right] \\ &\text{with } \Delta = b_U(1+\Gamma_{Uc})((b_D(1+\Gamma_{Dc}) + \theta^2 b_U \Gamma_{Uc}) - (\theta b_U \Gamma_{Uc})^2 \end{aligned}$$

Assumptions 1 are satisfied under the following conditions on parameters: First, there is no clean production initially if $p_U^0 < c_{Uc}$ and $p_D^0 < c_{Dc} + \theta p_U^0$. Second, a fully clean situation with positive production and consumption of both goods exists if and only if

$$\left[\theta + \frac{b_D(1+\Gamma_{Dc})}{\theta b_U \Gamma_{Uc}}\right](a_U - c_{Uc}) > a_D - c_{Dc} - \theta c_{Uc} > \theta \frac{\Gamma_{Uc}}{1+\Gamma_{Uc}}(a_U - c_{Uc}).$$

The first inequality ensures a non negative consumption upstream and the second a non negative consumption downstream. The clean upstream production should be sufficiently abundant, Γ_{Uc} small, to serve both markets (both extreme expressions are equal for $\Gamma_{Uc} = +\infty$). And the relative size of the upstream sector (b_D/b_U) should be sufficiently large to ensure that the downstream sector does not completely absorb the clean upstream production.

B Proofs

B.1 Proof of Proposition 1

We provide the proof for the case in which all technologies are used ($q_{ij}^* > 0$), other cases in which a technology is not used can be obtained as a specification of that case.

Equilibrium quantities are functions of the four instruments $q_{ij}^E(t_D, s_D, t_U, s_U)$ for $i, j \in \{U, D\} \times \{c, d\}$, the equilibrium upstream price is $p_U^E(t_D, s_D, t_U, s_U)$. These functions satisfy the four equations (9a), (9b), (9c).

It is useful to derive some general expressions of the change of quantities on each market with respect to an instrument $\tau \in \{t_D, s_D, t_U, s_U\}$. From the equilibrium on the dowsntream market , taking the full derivative of the couple of equations $P_D(q_{Dd}^E + q_{Dc}^E) = C'_{Dd}(q_{Dd}) + t_D = C'_{Dc}(q_{Dc}) - s_D + \theta p_U^E$ with respect to τ, using the definition of Γ_{Uj} (equation (4a)), and the Kronecker delta ($\delta_{ij} = 1$ if $i = j$ and 0 otherwise), gives

$$\begin{bmatrix} \frac{\partial q_{Dd}^E}{\partial \tau} \\ \frac{\partial q_{Dc}^E}{\partial \tau} \end{bmatrix} = \frac{1}{(-P'_D)(\Gamma_{Dd}\Gamma_{Dc} + \Gamma_{Dd} + \Gamma_{Dc})} \begin{bmatrix} 1+\Gamma_{Dc} & -1 \\ -1 & 1+\Gamma_{Dd} \end{bmatrix} \begin{bmatrix} -\delta_{t_D,\tau} \\ \delta_{s_D,\tau} - \theta \frac{\partial p_U^E}{\partial \tau} \end{bmatrix} \tag{26}$$

Similarly, on the upstream market, derivation of the couple of equations $P_U(q_{Ud}^E + q_{Uc}^E - \theta q_{Dc}) = C'_{Ud}(q_{Ud}) + t_U = C'_{Uc}(q_{Uc}) - s_U$ gives:

$$\begin{bmatrix} \frac{\partial q_{Ud}^E}{\partial \tau} \\ \frac{\partial q_{Uc}^E}{\partial \tau} \end{bmatrix} = \frac{1}{\Gamma_{Ud}\Gamma_{Uc} + \Gamma_{Ud} + \Gamma_{Uc}} \begin{bmatrix} 1+\Gamma_{Uc} & -1 \\ -1 & 1+\Gamma_{Ud} \end{bmatrix} \begin{bmatrix} -\frac{\delta_{t_U,\tau}}{-P'_U} + \theta \frac{\partial q_{Dc}^E}{\partial \tau} \\ \frac{\delta_{s_U,\tau}}{-P'_U} + \theta \frac{\partial q_{Dc}^E}{\partial \tau} \end{bmatrix} \tag{27}$$

Downstream instruments:

The consequences of downstream regulations on the upstream quantities are mediated through the quantity of the clean downstream, from equations (27):

$$\frac{\partial q_{Uj}^E}{\partial \tau} = \frac{\theta/\Gamma_{Uj}}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \frac{\partial q_{Dc}^E}{\partial \tau} \tag{28}$$

and

$$\frac{\partial p_U^E}{\partial \tau} = \frac{-\theta P_U'}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \frac{\partial q_{Dc}^E}{\partial \tau} \tag{29}$$

Let us now look at each downstream instrument in turn.

- Downstrem tax t_D: the principle of the proof is to write the changes of other quantities as functions of the change of q_{Dd}^E.

  From the first order equation (9b)

  the impact of the downstream tax on the quantity of clean downstream is such that (from equation (26))

$$\frac{\partial q_{Dc}^E}{\partial t_D} = \frac{-1}{1 + \Gamma_{Dc}} \left[\frac{\partial q_{Dd}^E}{\partial t_D} + \frac{\theta}{-P_D'} \frac{\partial p_U^E}{\partial t_D} \right];$$

  injecting into equation (29), the change of the upstream price as a function of the change of the dirty downstream is

$$\frac{\partial p_U^E}{\partial \tau} = -\frac{\partial q_{Dd}^E}{\partial t_D} \left[\frac{-1}{\theta P_U'} \left(1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}} \right) (1 + \Gamma_{Dc}) + \frac{\theta}{-P_D'} \right]^{-1}.$$

  From equation (26), we get that $\partial q_{Dd}^E / \partial t_D$ is negative because $\partial p_U^E / \partial \tau$ is negatively related to it.

  So, a small increase of t_D leads to a change of emissions equal to

$$\begin{aligned} &\frac{\partial q_{Dd}^E}{\partial t_D} \times \left\{ -\alpha_D + \alpha_U \frac{1}{-P_U' \Gamma_{Ud}} \left[\frac{-1}{\theta P_U'} \left(1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}} \right) (1 + \Gamma_{Dc}) + \frac{\theta}{-P_D'} \right]^{-1} \right\} \\ &= -\alpha_D + \theta \alpha_U \left[\left(1 + \Gamma_{Ud} + \frac{\Gamma_{Ud}}{\Gamma_{Uc}} \right) (1 + \Gamma_{Dc}) + \Gamma_{Ud} \frac{\theta^2 P_U'}{P_D'} \right]^{-1} \\ &\leq -\alpha_D + \theta \alpha_U \leq 0 \end{aligned}$$

- Downstream subsidy:

  From equation (9a)

$$\frac{\partial q_{Dd}^E}{\partial s_D} = -\frac{1}{1 + \Gamma_{Dd}} \frac{\partial q_{Dc}^E}{\partial s_D} \tag{30}$$

  and, $\partial q_{Dc}^E / \partial s_D$ is positive from equations (26) and (29). Then, from equation (28) the effect of s_D on total emissions is

$$\frac{\partial q_{Dc}^E}{\partial s_D} \left\{ -\frac{\alpha_D}{1 + \Gamma_{Dd}} + \alpha_U \frac{\theta / \Gamma_{Uj}}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \right\},$$

  condition (10) follows.

Upstream instruments: Upstream instruments will influence downstream quantities through the upstream price as described by equations (26).

- Upstream tax: we write all changes as a function of the upstream dirty quantity change.

  Let us denote ϕ the slope of the clean downstream with respect to the upstream price :

$$\phi = -\frac{\partial q_{Dc}^E/\partial t_D}{\partial p_U^E/\partial t_D} = \frac{\theta(1+\Gamma_{Dd})}{(-P_D')(\Gamma_{Dd}\Gamma_{Dc}+\Gamma_{Dd}+\Gamma_{Dc})}.$$

  The change of the upstream price solves:

$$\begin{aligned}\frac{\partial p_U^E}{\partial t_D} &= P_U' \times \left(\frac{\partial q_{Ud}^E}{\partial t_U} + \frac{\partial q_{Uc}^E}{\partial t_U} - \theta\frac{\partial q_{Dc}^E}{\partial t_U}\right) = P_U'\frac{\partial q_{Ud}^E}{\partial t_U} - \frac{1}{\Gamma_{Uc}}\frac{\partial p_U^E}{\partial t_U} + \theta P_U'\phi\frac{\partial p_U^E}{\partial t_U} \\ &= P_U'\frac{\partial q_{Ud}^E}{\partial t_U} \times \left[1 + \frac{1}{\Gamma_{Uc}} - \theta P_U'\phi\right]^{-1}\end{aligned}$$

  then, indeed $\partial q_{Ud}^E/\partial t_U \leq 0$ and, using $\partial q_{Dd}^E/\partial t_U = \phi(1+\Gamma_{Dd})(\partial p_U^E/\partial t_U)$, the change of emissions is

$$\frac{\partial q_{Ud}^E}{\partial t_D}\left\{\alpha_U + \alpha_D\frac{\phi}{1+\Gamma_{Dd}}\frac{P_U'}{1+\Gamma_{Uc}-\theta P_U'\phi}\right\} = \frac{\partial q_{Ud}^E}{\partial t_D}\left\{\alpha_U - \frac{\alpha_D}{1+\Gamma_{Dd}}\frac{1}{(1+\frac{1}{\Gamma_{Uc}})/(-\phi P_U') + \theta}\right\}$$

  and injecting the expression of ϕ gives condition (11).

- Upstream subsidy:

  the upstream price derivative is

$$\begin{aligned}\frac{\partial p_U^E}{\partial S_U} &= P_U' \times \left(\frac{\partial q_{Ud}^E}{\partial s_U} + \frac{\partial q_{Uc}^E}{\partial s_U} - \theta\frac{\partial q_{Dc}^E}{\partial s_U}\right) = P_U'\frac{\partial q_{Uc}^E}{\partial s_U} - \frac{1}{\Gamma_{Ud}}\frac{\partial p_U^E}{\partial s_U} + \theta P_U'\phi\frac{\partial p_U^E}{\partial s_U} \\ &= P_U'\frac{\partial q_{Uc}^E}{\partial s_U} \times \left[1 + \frac{1}{\Gamma_{Ud}} - \theta P_U'\phi\right]^{-1}\end{aligned}$$

  then, $\partial q_{Uc}/\partial s_U \geq 0$ and $\partial p_U^E/\partial s_U \leq 0$, so the change of emissions:

$$\alpha_D\phi(1+\Gamma_{Dd})\frac{\partial p_U^E}{\partial S_U} + \alpha_U\frac{1}{C_{Dd}''}\frac{\partial p_U^E}{\partial S_U} \leq 0$$

B.2 Proof of Corollary 1

The optimal allocation corresponds to the market equilibrium with $t_i = \alpha_i\mu$ and $s_i = 0$, in sector $i = U, D$. The Corollary can then be proved by using above calculations with

$$\frac{dq_{ij}^*}{d\mu} = \alpha_D\frac{\partial q_{ij}^E}{dt_D} + \alpha_U\frac{\partial q_{ij}^E}{dt_D}.$$

A more elegant way to proceed, is to isolate one good $(i,j) \in \{U,D\} \times \{d,c\}$, and define the economic benefit as a function of E and q_{ij}:

$$B(E, q_{ij}) = \max_{q_{kl},(k,l)\neq(i,j)} \{W(\mathbf{q}) + \mu E | \alpha_D q_{Dd} + \alpha_U q_{Ud} \leq E\}.$$

Indeed $E = \alpha_D q_{Dd} + \alpha_U q_{Ud}$ must be larger than $\alpha_i q_{ij}$ if $j = d$. The optimal allocation E^* and q^*_{ij} then solves

$$\frac{\partial B}{\partial E} = \mu \text{ and } \frac{\partial B}{\partial q_{ij}} = 0$$

and an increase of the SCC μ leads to an increase of q^*_{ij} if and only if

$$\frac{\partial^2 B}{\partial E \partial q_{ij}} \leq 0,$$

which means that emissions and the good ij are substitutes.

The optimal allocation is decentralized with $t_U = \alpha_U \mu$, $t_D = \alpha_D \mu$, and $s_D = s_U = 0$. With an additional tax t on good ij (on top of $\alpha_i t_i$ if $j = d$) equilibrium emissions E^E, which is equal to $\sum_i \alpha_i q^E_{id}$, and quantity q^E_{ij} solve

$$\frac{\partial B}{\partial E} = \mu \text{ and } \frac{\partial B}{\partial q_{ij}} = t;$$

an increase of t (keeping μ constant and thus the three other instruments) increases emissions if and only if $\partial^2 B / \partial E \partial q_{ij} \leq 0$. Therefore, we can state that

Result The quantity q^*_{ij} decreases with respect to the SCC, if and only if, emissions are decreasing with respect to τ_i at the Pigovian regulation $(t_D, t_U, s_D, s_U) = (\alpha_D \mu, \alpha_U \mu, 0, 0)$, with $\tau_i = t_i$ if $j = d$ and $t_i = -s_j$ if $j = c$.

The corollary then follows from Proposition 1.

B.3 Proof of propositions 2 and 4

Proposition 2: The optimal downstream subsidy solves equation (13), with $\tau = s_D$. The derivatives of each quantity with respect to s_D are given by equation (30) for the dowsntream dirty quantity and by equation (28) for upstream quantities. Formula (14) follows.

Proposition 4: The couple of optimal subsidies solves two equations (13), with $\tau = s_D$ and $\tau = s_U$. From the market equilibrium conditions (9a), dirty quantities change are given by, for $i = U, D$:

$$\frac{\partial q^E_{Dd}}{\partial s_i} = -\frac{1}{1+\Gamma_{Dd}} \frac{\partial q^E_{Dc}}{\partial s_i} \text{ and } \frac{\partial q^E_{Ud}}{\partial s_i} = \frac{1}{1+\Gamma_{Ud}} \left[\frac{\partial q^E_{Uc}}{\partial s_i} - \theta \frac{\partial q^E_{Dc}}{\partial s_i} \right].$$

Injecting these two equations into equation (13), gives

$$\left[s_U - \frac{\alpha_U \mu - t_U}{1+\Gamma_{Ud}} \right] \frac{\partial q^E_{Uc}}{\partial s_i} + \left[s_D - \frac{\alpha_D \mu - t_D}{1+\Gamma_{Dd}} + \theta \frac{\alpha_U \mu - t_U}{1+\Gamma_{Ud}} \right] \frac{\partial q^E_{Dc}}{\partial s_i} = 0$$

for $i = U, D$, the two expressions (18) solve these first order conditions.

B.4 Proof of Corollary 2

With a mandate r_U, denoting q the total upstream production, $q^r_{Uc}(p_U) = r_U q(p_U)$, $q^r_{Ud}(p_U) = (1-r_U)q(p_U)$ and $q(p_U)$ solves

$$p_U = r_U C'_{Uc}(r_U q) + (1-r_U)C'_{Ud}((1-r_U)q).$$

Therefore, $q^{r\prime}_{Uc}(p_U) = r_U q'(p_U)$, $q^{r\prime}_{Ud}(p_U) = (1-r_U)q'(p_U)$, and the Γ^r_{Uj} are then:

$$\frac{1}{\Gamma^r_{Uc}} = r_u(-P'_U)q', \text{ and } \frac{1}{\Gamma^r_{Ud}} = (1-r_u)(-P'_U)q'.$$

Injecting the above expressions into formula (16):

$$\begin{aligned} s_D &= \frac{\alpha_D\mu - t_D}{1+\Gamma_{Dd}} - \frac{\theta(-P'_U)q'}{1-q'P'_U}\Big[\big(\alpha_U\mu - r_u\left(C'_{Uc}-C'_{Ud}\right)\big)(1-r_u) + (1-r_U)\big(C'_{Uc}-C'_{Ud}\big)r_U\Big] \\ &= \frac{\alpha_D\mu - t_D}{1+\Gamma_{Dd}} - \frac{\theta\alpha_U\mu(1-r_u)}{1+\frac{1}{(-P'_U)q'}} \end{aligned}$$

And replacing $q' = [r_U^2 C'_{Uc} + (1-r_U)^2 C'_{Ud}]^{-1}$ (from the upstream suppliers first-order conditions) gives formula (17).

B.5 Proof of Lemma 4

In Proposition 4 dirty quantities were supposed positive. However, for large μ they are null and the subsidies are used to keep them null. Formally, the welfare function is not continuously differentiable everywhere with respect to instruments because of corners situations in which one of the q_{ij} is null. Concerning q_{Ud}, it is null if $p_U \leq t_U + C'_{Ud}(0)$, that is, if q^E_{Uc} and q^E_{Dc} are such that

$$P_U(q^E_{Uc} - \theta q^E_{Dc}) \leq t_U + C'_{Ud}(0).$$

For s_U, s_D such that $p_U < t_U + C'_{Ud}(0)$ the derivative of welfare with respect to s_i is

$$-s_U\frac{\partial q_{Uc}}{\partial s_i} - \left[s_D - \frac{\alpha_D\mu - t_D}{1+\Gamma_{Dd}}\right]\frac{\partial q_{Dc}}{\partial s_i}$$

and the couple (s_U, s_D) that cancels these equation implies $p_U > t_U + C'_{Ud}(0)$, so that, if $q^E_{Ud} = 0$ at the second best then welfare is maximized along the boundary $P_U(q^E_{Uc} - \theta q^E_{Dc}) = t_U + C'_{Ud}(0)$. So s_U, s_D maximize

$$W + \lambda[q^E_{Uc} - \theta q^E_{Dc} - P_U^{-1}(t_U + C'_{Ud}(0))]$$

for some $\lambda > 0$. The optimality conditions are then

$$(\lambda - s_U)\frac{\partial q_{Uc}}{\partial s_i} - \left[s_D - \frac{\alpha_D\mu - t_D}{1+\Gamma_{Dd}} - \theta\lambda\right]\frac{\partial q_{Dc}}{\partial s_i} = 0 \text{ for } i = U, D$$

$$q^E_{Uc} - \theta q^E_{Dc} = P_U^{-1}(t_U + C'_{Ud}(0)).$$

Therefore, $\lambda = s_U$ and s_D is given by equation (19). The two subsidies are such that $p^E_U = t_U + C'_{Ud}(0)$ and since, at the marekt equilibrium, $s_U = C'_{Uc}(q^E_{Uc}) - p^E_U$ equation (20) follows.

B.6 Proof of Corollary 3

The exposition of the proof is lighter by rewriting welfare as a function of difference between actual and optimal quantities. Let us define

$$z_1 = q_{Dc} - q_{Dc}^{FB},\ z_2 = q_{Uc} - q_{Uc}^{FB},\ z_3 = q_{Dd} - q_{Dd}^{FB},\ z_4 = q_{Ud} - q_{Ud}^{FB},$$

Rewrite welfare as:

$$W(z_1, z_2, z_3, z_4) = W^{FB} - \sum_i \frac{\gamma_i}{2} z_i^2 - \sum_{j \neq i} \frac{\gamma_{ij}}{2} z_i z_j$$

with $\gamma_{ij} = \gamma_{ji}$ (it is straightforward to write the γs as functions of the parameters of Specification 1 but not necessary for the proof, the result holds more generally).

Quantities The optimal first-best quantities are $z_i^{FB} = 0$. Our second-best case corresponds to a situation in which there are two subsidies on quantities 3 and 4 to be denoted σ_3 and σ_4 ($\sigma_3 = \alpha_D \mu - t_D$ and $\sigma_4 = \alpha_U \mu - t_U$), and welfare is optimized with respect to z_1 and z_2.[18] The two quantities z_3 and z_4 depends on z_1 and z_2 and solve

$$\frac{\partial W}{\partial z_i}(z_1, e_2, z_3, z_4) = -\sigma_i \text{ for } i = 3, 4$$

which gives

$$\begin{bmatrix} \gamma_3 & \gamma_{34} \\ \gamma_{34} & \gamma_4 \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} \sigma_3 - \gamma_{13} z_1 - \gamma_{23} z_2 \\ \sigma_4 - \gamma_{14} z_1 - \gamma_{24} z_2 \end{bmatrix}$$

inverting the matrix, denoting $\delta = \gamma_3 \gamma_4 - \gamma_{34}^2$:

$$\begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \frac{1}{\delta} \begin{bmatrix} \gamma_4 (\sigma_3 - \gamma_{13} z_1 - \gamma_{23} z_2) - \gamma_{34} (\sigma_4 - \gamma_{14} z_1 - \gamma_{24} z_2) \\ \gamma_3 (\sigma_4 - \gamma_{14} z_1 - \gamma_{24} z_2) - \gamma_{34} (\sigma_3 - \gamma_{13} z_1 - \gamma_{23} z_2) \end{bmatrix}.$$

The second-best z_1 solves

$$0 = \frac{\partial W}{\partial z_1} + \frac{\partial W}{\partial z_3}\frac{\partial z_3}{\partial z_1} + \frac{\partial W}{\partial z_4}\frac{\partial z_4}{\partial z_1} = -\left[\gamma_1 z_1 + \gamma_{12} z_2 + \gamma_{13} z_3 + \gamma_{14} z_4\right] - \sum_{i=3,4} \sigma_i \frac{\partial z_i}{\partial z_1}.$$

The last term is the subsitution effect mentioned in the main text and is equal to

$$-\sigma_3 \frac{(-\gamma_4 \gamma_{13} + \gamma_{34} \gamma_{14})}{\delta} - \sigma_4 \frac{(-\gamma_3 \gamma_{14} + \gamma_{34} \gamma_{13})}{\delta}.$$

The first bracketed term depends on z_3 and z_4, this is the substitution effect mentioned in the main text. The two quantities z_3 and z_4 are linear in σ_3 and σ_4 with

$$\frac{\partial}{\partial \sigma_3}(\gamma_{13} z_3 + \gamma_{14} z_4) = \frac{1}{\delta}\left[\gamma_{13}\gamma_4 - \gamma_{14}\gamma_{34}\right]$$

[18] Any subsidy couple (σ_1, σ_2) is associated with a unique quantity couple (x_1, x_2), and vice-versa; the two other quantities z_3 and z_4 can indifferently be written as function of the former or the latter.

Therefore, the terms in σ_3 and σ_4 cancel out and the first-order condition above simplifies to a linear equation with a null fixed term:

$$0 = -\gamma_1 z_1 - \gamma_{12} z_2 - \frac{\gamma_{13}}{\delta}\left[(\gamma_{34}\gamma_{14} - \gamma_4\gamma_{13})z_1 + (\gamma_{34}\gamma_{24} - \gamma_4\gamma_{23})z_2\right] - \frac{\gamma_{14}}{\delta}\left[(\gamma_{34}\gamma_{13} - \gamma_3\gamma_{14})z_1 + (\gamma_{34}\gamma_{23} - \gamma_4\gamma_{24})z_2\right]$$

following the same steps for z_2 gives another linear equation with a null fixed term. The optimal second-best solution is then $z_1 = z_2 = 0 = z_1^{FB} = z_2^{FB}$.

Welfare comparison At the second-best welfare is

$$W = W^{FB} - \frac{\gamma_3}{2} z_3^2 - -\frac{\gamma_4}{2} z_4^2 - \gamma_{34} z_3 z_4.$$

And, from Specification 1, the coefficients are $\gamma_{34} = \partial^2 W/\partial q_{Ud} q_{Dd} = 0$, $\gamma_3 = b_D(1 + \Gamma_{Dd})$ and $\gamma_4 = b_U(1 + \Gamma_{Ud})$. Therefore, $z_3 = \gamma_4\sigma_3/\delta = \sigma_3/\gamma_3$, and $z_4 = \sigma_4/\gamma_4$. Welfare is

$$W = W^{FB} - \frac{\sigma_3^2}{2\gamma_3} - \frac{\sigma_4^2}{2\gamma_4} = W^{FB} - \frac{(\alpha_D\mu - t_D)^2}{2b_D(1+\Gamma_{Dd})} - \frac{(\alpha_U\mu - t_U)^2}{2b_U(1+\Gamma_{Ud})}.$$

Sectoral welfares We use our specificaton 1. Since $q_{ic}^{FB} = q_{ic}^{SB}$ we have $P_U^{FB} - P_U^{SB} = s_U^{SB}$ (from market equilibrium condition (9c)) and $q_{id}^{FB} - q_{id}^{SB} = Q_i^{FB} - Q_i^{SB}$. The difference of gross consumer surplus can be written

$$S_i^{FB} - S_i^{SB} = (Q_i^{FB} - Q_i^{SB})[a_i - \frac{b_i}{2}(Q^{FB} + Q^{SB})] = \frac{1}{2}(Q_i^{FB} - Q_i^{SB})(p_i^{FB} - p_i^{SB})$$

and the difference of dirty production costs (with a similar manipulation):

$$C_{id}^{FB} - C_{id}^{SB} = \frac{1}{2}(q_{id}^{FB} - q_{id}^{SB})\left[C'_{id}(q_{id}^{FB}) + C'_{id}(q_{id}^{SB})\right] = \frac{1}{2}(Q_i^{FB} - Q_i^{SB})\left[(p_i^{FB} - \alpha_i\mu) - (p_i^{SB} - t_i)\right].$$

The difference of welfare between FB and SB is then (clean production costs cancel out):

$$W_U^{FB} - W_U^{SB} = S_U^{FB} - S_U^{SB} - \left[C_{Ud}^{FB} - C_{Ud}^{SB}\right] + \theta(p_U^{FB} - p_U^{SB})q_{Dc} \quad (31)$$

$$= \frac{1}{2}\left(Q_U^{SB} - Q_U^{FB}\right)\left(\alpha_U\mu - t_U\right) + \theta s_U^{SB} q_{Dc} \quad (32)$$

and

$$W_D^{FB} - W_D^{SB} = -\theta s_U^{SB} q_{Dc} + \frac{1}{2}\left(Q_D^{SB} - Q_D^{SB}\right)\left(\alpha_D\mu - t_D\right).$$

And $b(Q_i^{SB} - Q_i^{FB}) = p_i^{FB} - p_i^{SB} = b_i\Gamma_{id}(q_{id}^{FB} - q_{id}^{SB}) + \alpha_i\mu - t_i$ Hence $Q_i^{SB} - Q_i^{FB} = \frac{1}{b_i(1+\Gamma_{id})}\left(\alpha_i\mu - t_i\right)$, expression (22) follows.

C Calibration

	Upstream: power sector		Downstream: passenger road transport	
	Value	Source	Value	Source
Q_i^0	473 TWh	RTE[a]	436 10^6 km	French Ministry of Ecology[b]
ϵ_i	0.8	INSEE[c]	0.7	Graham and Glaister (2004)
c_{id}	177 €/MWh	RTE[a]	0.5 €/km	French Ministry of the Economy[d]
α_i	0.350 tCO_2/MWh	RTE[a]	0.120 kCO_2/km	ADEME[e]
θ			0.2 kWh/km	ADEME[e]

a: https://www.rte-france.com/en/eco2mix
b: https://www.statistiques.developpement-durable.gouv.fr/chiffres-cles-du-transport-edition-2019
c: https://www.insee.fr/fr/statistiques/4467133?sommaire=4467460
d: https://www.economie.gouv.fr/particuliers/bareme-kilometrique
e: https://carlabelling.ademe.fr/

Table 5: Parameter values and sources

Our numerical illustration is based on the electrification of passenger car in France. The upstream sector is then the power sector and the downstream is the transport sector restricted to passenger cars. We use several data source from different official agencies and from the academic literature. In each sector, demand and costs are calibrated using values from 2019. Parameters values and sources are given in table 5. To calibrate the demand function we proceed as follow: from a BAU price equal to the dirty marginal cost c_{id}, a BAU quantity Q_i^0 and a price-elasticity of demand ϵ_i in sector $i = U, D$, we infer a_i and b_i from

$$a_i = c_{id}\big(1 + \frac{1}{\epsilon_i}\big)$$
$$b_i = \frac{c_{id}}{Q_i^0 \epsilon_i}$$

We did not find any available data source to easily calibrate c_{ic} et Γ_{ic}. Hence we choose two SCCs μ_i^0 at which the clean technology starts being competitive:

$$c_{Uc} = c_{ed} + \alpha_U \mu_U^0$$
$$c_{Dc} = c_{md} - \theta c_{ed} + (\alpha_U - \theta \alpha_D)\mu_D^0$$

We choose $\mu_U^0 = 10$€ / tCO_2 and $\mu_D^0 = 50$€ / tCO_2.

Then, we choose two μ_i^1 together with a share z_i of the clean technology in sector i such that $q_{ic}(\mu_i^1) = z_i Q_i^0$ (μ_i^1 is not too large to ensure that the q_{jd} are positive). We derive the

Γ_{ic} from the formula:

$$
\begin{aligned}
c_{Uc} &= c_{ed} + \alpha_U \mu_U^0 \\
c_{Dc} &= c_{md} - \theta c_{ed} + (\alpha_U - \theta \alpha_D)\mu_D^0 \\
\Gamma_{Uc} &= b_U \frac{\alpha_U(\mu_U^1 - \mu_U^0)}{z_U^1 Q_U^0} \\
\Gamma_{Dc} &= b_D \frac{\alpha_U(\mu_D^1 - \mu_D^0)}{z_D^1 Q_D^0}
\end{aligned}
$$

We choose $\mu_D^1 = 300$ tCO_2 and $z_D^1 = 0.5$ for the downstream sector and $\mu_U^1 = 200 tCO_2$ and $z_U = 0.8$ for the upstream sector.